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# The Friction Factor of Turbulent Flow of the Base Fluid and Nanofluid in Statistical Approach

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## 1. Introduction

One of the most important parameters that is always considered in fluid transfer is the friction factor. The friction factor in the fluid flow depends on various variables such as Reynolds number, flow structure in terms of laminar or turbulence, the volume fraction of nanoparticles, size and shape of nanoparticles, channel dimensions and Prandtl number. The aim of this study was to investigate the equations of friction factor in turbulent flow of base fluids and nanofluids and statistical analysis of this parameter. Nanofluids are a group of heat transfer fluids in which metallic and non-metallic particles are suspended in dimensions of less than 100 nm and significantly increase the heat transfer potential. Nanofluids are used to improve and increase heat transfer instead of base fluids such as water and ethylene glycol. Researchers studying nanofluids usually consider two issues. The first is how much heat transfer increases with the use of nanofluids, and the second is how much the pressure drop due to the use of nanofluids increases. At the end of the study, based on experimental or numerical data, they present equations for the Nusselt number and the friction factor. Based on the developed equations of friction factor, it is observed that fluid transfer systems and heat transfer facilities include uncertainties. These uncertainties are in fact the same independent variables including Reynolds number, volume fraction, and nanoparticle diameter, flow geometry, and so on. Therefore, changing any variable leads to a change in the friction factor and pressure drop. Accordingly, statistical analysis of dependent variables such as friction factor is necessary for proper system efficiency.

In this study, first the presented friction factor equations for the base fluid and nanofluid in the turbulent flow were considered. Then, due to the importance of Reynolds number in the turbulence flow based on the change of this variable, statistical analysis was performed. Easy Fit software was used for the statistical analysis. In order to select the best probability distribution function of the dependent variable friction factor, 48 different probability distribution functions were used using Kolmogorov-Smirnov, Anderson-Darling and chi squared tests. After reviewing the results, it was found that based on all three statistics, the probability distribution function of the friction factor in the base fluid and nanofluid is the Johnson SB probability distribution function. On the other side, it was observed that the probability distribution function of the friction factor of the base fluid and nanofluid equations is wider than the normal distribution function and the probability distribution function of Johnson SB also has a right skewness.

## 2. The presented equations for the base fluid and their statistical analysis

After reviewing various studies in the field of nanofluids, it was observed that most researchers often use the two equations of Blasius and Petukhov to compare the numerical and experimental results of the friction factor and ensure the accuracy of the conducted study. These equations depend only on single variable that is the Reynolds number. The two equations of Naphon et al. and Li et al. developed in terms of Reynolds numbers, have also been used for statistical analysis that the latter being expressed for rough tubes. To determine the best probability distribution function for the friction factor, 48 different multiparameter distribution functions have been used. After reviewing the results based on Kolmogorov-Smirnov, Anderson-Darling and chi squared statistics, it was found that the best probability distribution function for the friction factor in the base fluid by changing the Reynolds number based on all three statistics is the Johnson SB probability distribution function. This probability distribution function is defined as follows:

$$f\left(f\right)=\frac{\delta}{\lambda\sqrt{2\pi}z\left(1-z\right)}\exp\left(-\frac{1}{2}\left(\gamma+\delta\ln\left(\frac{z}{1-z}\right)\right)^{2}\right) \qquad (1)$$

In equation (1), $z=(f-\xi)/\lambda$, $\gamma$ and $\delta$ are the continuous shape parameters, $\lambda$ is the continuous scale parameter and $\xi$ is the continuous location parameter. The Johnson SB probability distribution function is plotted in Fig. 1 for the friction factor's data of Blasius, Petukhov, Naphon et al., and Li et al. equations.

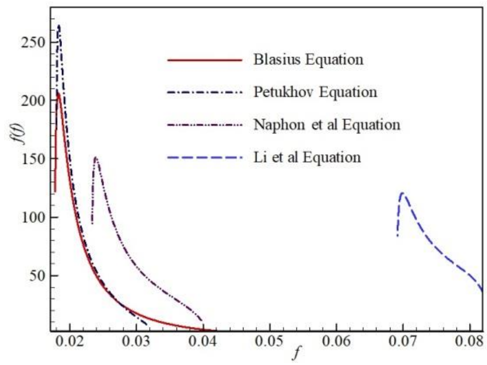


**Figure 1. Johnson SB probability distribution function of friction factor for Blasius, Petukhov, Naphon et al., and Li et al. equations**

Figure 1. shows that the probability distribution function of the friction factor of all equations has a right skewness.

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## 3. Statistical analysis of the equations of the friction factor of the nanofluid flow with single variable

The equations of the friction factor of the nanofluid flow with single variable are in fact only in terms of Reynolds numbers. These equations with other variables held constant as the volume fraction of nanoparticles have been developed. In this case, the probability distribution function of the data of the friction factor extracted from the equations, Johnson SB, was concluded. Figure 2 demonstrates the probability distribution function of Johnson SB for friction factor of the three nanofluids.

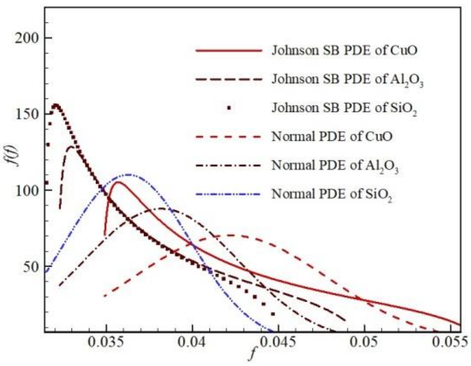


**Figure 2. Johnson SB and Normal probability distribution function of friction factor for presented equations by Vajjha et al.**

It is observed that the data accumulation on the left side of the graphs is more than that of the right side, and in fact in the high Reynolds numbers the changes in the friction factor are less. On the other hand, it is observed that for all three nanofluids the Johnson SB probability distribution function is wider than the normal probability distribution function.

## 4. Statistical analysis of the equations of the friction factor of the nanofluid flow with two variables

The two variable equations are developed in terms of Reynolds number and volume fraction of nanoparticles. After reviewing the results, it was observed that the best probability distribution function of the friction factor of two variable equations, like single variable equations, is the Johnson SB probability distribution function (Figure 3).

## 5. Statistical analysis of the equations of the friction factor of the nanofluid flow with three variables and more

These equations have been developed according to different variables such as Reynolds number, volume fraction of nanoparticles, ratio of nanoparticle diameter to diameter particle of base fluid, volume fraction of each particle in hybrid nanofluids, aspect ratio or width to channel height ratio, hydraulic diameter ratio to the inner diameter of the channel in the channels containing the tape and the rotation ratio of the tape.

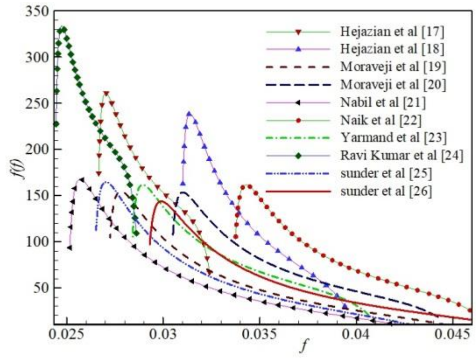


**Figure 3. Johnson SB probability distribution function of friction factor for two variables of presented equations of nanofluid**

In this section, as in previous cases, after analyzing the results, it was observed that the best probability distribution function of the friction factor based on all 3 statistics is the Johnson SB probability distribution function. Figure 4 demonstrates the diagram of the Johnson SB probability distribution function of friction factor for three variables or more of presented equations of nanofluid.

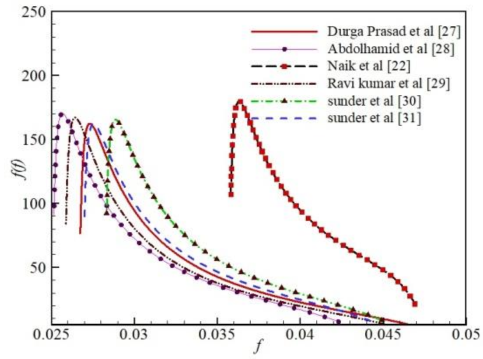


**Figure 4. Johnson SB probability distribution function of friction factor for three variables or more of presented equations of nanofluid**

## 6. Conclusion

In the present study, the equations of friction factor presented by researchers in base fluid and various nanofluids have been statistically analyzed. The intended equations are selected based on the turbulent fluid flow. 48 probability distribution functions have been used in statistical analysis. The Kolmogorov-Smirnov, Anderson-Darling and chi square statistics were used to determine the best probability distribution function for the friction factor's data. After reviewing and analyzing the results, it was found that the best probability distribution function of the presented equations of friction factor in both the base fluid and the nanofluid, based on all 3 statistics, is Johnson SB probability distribution function.