Comparison of Monolithic and Partitioned Approaches for the Solution of Fluid-Solid Thermal Interaction Problems

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1. Introduction

Thermal interaction between fluid flow and solid, known as conjugate heat transfer (CHT), has been widely studied due to its industrial applications. This problem is classified as a multi-physics phenomenon which is characterized by at least two distinct computational sub-domains, i.e. fluid and solid regions, whose associated equations are coupled at shared boundaries. In addition to interface coupling, there are two other types of coupling; one is between momentum and energy in fluid region and another is known as pressure-velocity coupling. These three are shown in the fig. 1.

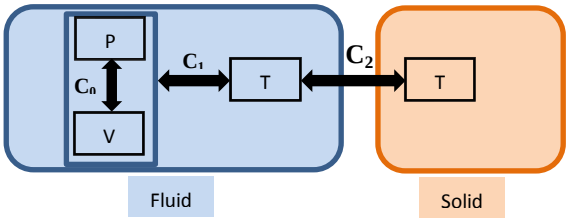

Figure 1. Three Types of coupling in CHT

Appropriate enforcement of coupling conditions at shared interfaces is one of the key effects on accuracy and computational costs in CHT problems especially in unsteady simulations. Based on previous studies, two types of coupling play a major role in stability and efficiency of CHT problems: coupling between energy and momentum equations in fluid as well as coupling associated with thermal interaction between two separate regions.

This study compares performance of three finite volume-based solvers all of which use a family of semi-implicit projection method[3], that is, PISO, to handle pressure-velocity coupling. However, they differ in dealing with two other types of coupling. In the monolithic approach, PISO algorithm with a single outer iteration is performed for solution of pressure and velocity in all fluid regions and it is followed by simultaneous solution of energy equation across all sub-domains; For partitioned methods, where energy equation for all sub-domains are solved in a sequent way, with two strategies: the first one is to apply separate iterative loop for enforcing each type of coupling, and the second suggests integrating two types of coupling in a single outer loop. Finally, the performance of the three algorithms is assessed via solution of thermal interaction between natural convection in an incompressible flow and conduction in vertical wall with different levels of coupling strength.

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2. Governing Equations

The time dependent equations for describing for a Boussinesq fluid flow, that is, continuity, momentum and energy equations, are expressed as follow

$$\nabla . u = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u.\nabla u = -\nabla p + \upsilon \nabla^2 u - g\beta(T - T_{ref}) \tag{2}$$

$$\frac{\partial T}{\partial t} + (u.\nabla)T = \frac{1}{\rho C_P}\nabla.(k\nabla T) \tag{3}$$

The time dependent energy equation for solid is

$$\frac{\partial T_s}{\partial t} = \frac{1}{(\rho C_P)_s}\nabla.(k_s \nabla T_s) \tag{4}$$

The temperature and heat flux must be continuous at shared interface, $\Gamma_{f,s}$ as follow

$$T(\Gamma_{f,s}, t) = T_s(\Gamma_{f,s}, t) \tag{5}$$

$$n.(k\nabla T) = n.(k_s \nabla T_s) \tag{6}$$

The coupling highly depends on the Grashof number, $Gr = \frac{g\beta(T_H - T_L)L^3}{\nu^2}$. The non-dimensional quantity for describing the strength of second type of coupling is:

$$\sigma = \frac{k_f/\sqrt{\alpha_f}}{k_s/\sqrt{\alpha_s}} \tag{7}$$

Here, α is thermal diffusivity. The interaction strength is weak for $\sigma << 1$and increases as $\sigma \to 1$.

3. Numerical Approaches

In this section, the numerical algorithms, which are developed in OpenFOAM package, are explained.

Algorithm (a), which is called semi-implicit projection method with separate loops (SIPM-SP), suggests an outer iteration for first type of coupling and a sub-iteration loop for second one. Algorithm (b), called semi-implicit projection method with integrated loop (SIPM-IP), integrates two loops into one. However, for monolithic solver, (SIPM-M), algorithm (c), there is only need for one iteration loop to strongly enforce first type of coupling. It is worth mentioning that algorithms (b) and (c) are implemented in standard solvers of OpenFOAM while algorithm (a) is developed by authors via modifying an existing partitioned solver in this software.

To evaluate k at interface, k_{int}, for monolithic approach, a harmonic interpolation is used:

$$k_{int} = \frac{k_f k_s}{\frac{\Delta_f}{\Delta} k_s + \frac{\Delta_s}{\Delta} k_f} \quad (8)$$

4. Results and Discussion

Numerical results for the case of 2D conjugate natural convection with thermal conduction in a thick vertical wall, shown in Figure 2, are presented to validate the accuracy of solvers. Figure 3 shows temperature profile at horizontal center line given by three algorithms and compares them with a reference.

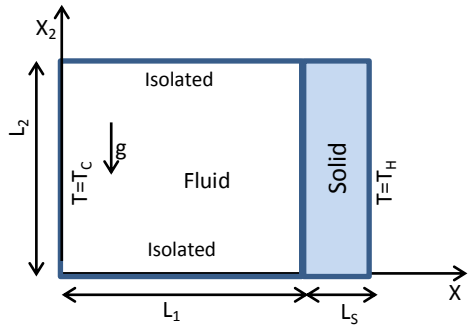

Figure 2. Natural convection adjacent to a thick vertical wall

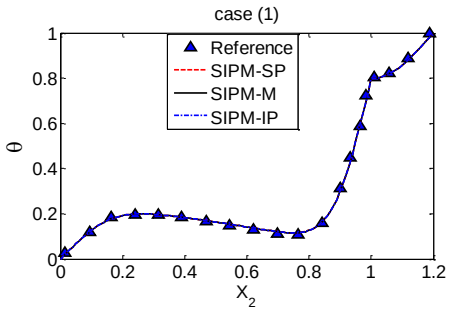

Figure 3. Temperature at horizontal line passing center

In order to evaluate the computational costs of three algorithms, four different cooling conditions are applied for the simulation of the test cases. Computational costs of the proposed algorithms for solution of the case A are compared in Figure 4. As seen in this figure, SIPM-IP produces the slowest convergence rate especially in case A and B, where strong thermal interaction exists between fluid and solid regions.

Table 1. Non-dimensional properties for four cases

	Pr	k_s/k_f	Gr	σ
CaseA	0.7	1.6	1.43×10^5	0.93
Case B	0.7	1.6	10^4	0.93
Case C	0.7	1600	1.43×10^5	4×10^{-4}
Case D	0.7	1600	10^4	4×10^{-4}

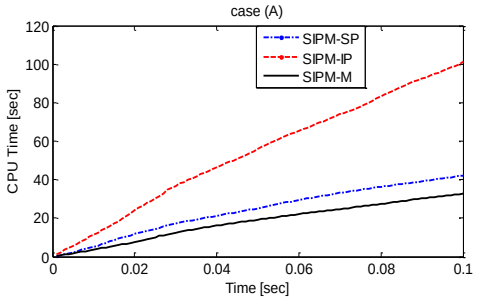

Figure 4. CPU time as a function of solution time

New test case with two regions of fluid and two regions of solid, shown in Figure 5, are also solved by three algorithms. Figure 6 shows the convergence results for case B of the new test case.

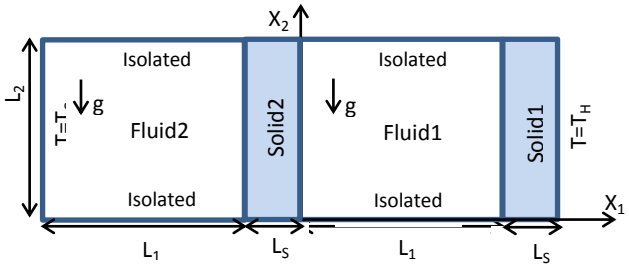

Figure 5. Multi-region test case

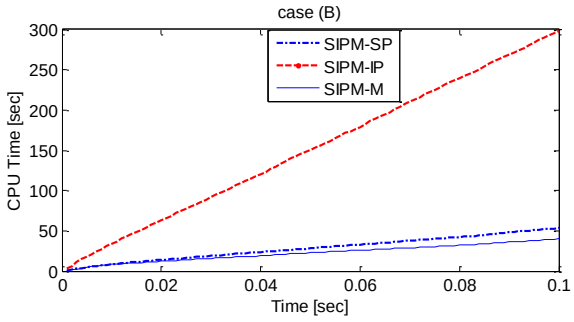

Figure 6- CPU time as a function of solution time

5. Conclusion

- Monolithic solution of energy equation across entire domain results in more efficient CHT solver.
- Comparing two types of partitioned methods, that is, SIPM-IP and SIPM-SP showed that employing separate iterative loop is advantageous in terms of efficiency especially where there is a strong coupling between energy equations at different sub-domains.