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The Solution of Nonlinear Compressible Hyperelastic Problems by the Isogeometric Analysis Method

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1. Introduction

Nonlinear behavior of materials, in both manufacturing and working environments is needed to be considered for optimal designing of engineering components. This can be achieved by numerical formulation and simulation of the governing equations that requires a good understanding of both theoretical background and associated computer solution techniques.

This paper aimed at the derivation of formulation and solution for nonlinear compressible elastic problems, known as compressible hyperelasticity, by the isogeometric analysis method. For this purpose, an algorithm is devised and by the linearization of governing equations, the discretized equilibrium equations are obtained. The matrix of coefficients is derived by using the isogeometric concept. Then, the obtained results by the proposed method for compressible hyperelastic materials is compared with the finite element method. Moreover, the effects of the number of Gauss integration points, as well as the number of load increments, on the convergence of the solution are studied.

2. Isogeometric Analysis in Compressible Hyperelasticity

The equilibrium equation in elasticity problems, both linear and nonlinear, is derived in terms of stresses inside the body. These stresses result from the deformation of material and it is necessary to express them in terms of deformation. These relationships, known as constitutive equations are established in the context of compressible hyperelastic materials, whereby stresses are derived from a stored elastic energy function. The relationship between stresses and stored elastic energy function is nonlinear and can be linearized within a potential Newton‑Raphson solution process. Moreover, the virtual work representation of the equilibrium equation presented is nonlinear with respect to the geometry. Hence, for a given material and loading conditions, the equilibrium equation is linearized and then discretized.

The discretization is established in the initial configuration to interpolate the initial geometry in terms of the control points and NURBS shape functions (Non Uniform Rational B-splines) in the framework of isogeometric analysis. The NURBS' basis functions are also employed for the purpose of approximation which plays a similar role as the shape functions in the finite element method. For construction of the solution surfaces for displacement components, choosing the position of the control points, an extra displacement component value is associated to them to be determined. Note that the mentioned imaginary surfaces belong to a space with an extra dimension with respect to the physical space where the domain of the problem has been defined.

3. Numerical Examples

In this section, to demonstrate the performance of the method, three examples including a 3D cantilever beam, a truncated cone and a bent thick plate problem is solved. Also, effects of the number of Gauss integration points and the number of load increments are studied.

Example 1. Three Dimensional Cantilever beam

The aim of this example is to compare the results of FEM and IGA. The geometry, loading, and boundary conditions for this example are illustrated in Figure 1 and the obtained results are depicted in Figures 2 and 3.

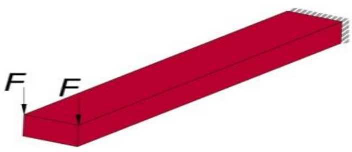
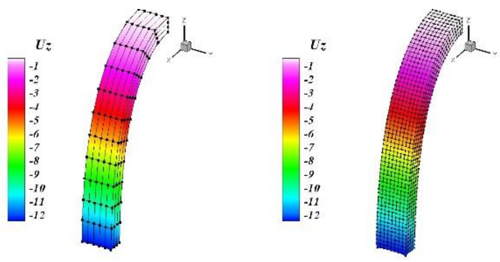


Fig. 1. Problem definition of 3D cantilever beam



(a) (b)

Fig. 2. Deformed shape and displacement contours in z direction for the IGA (left) and FEM (right)

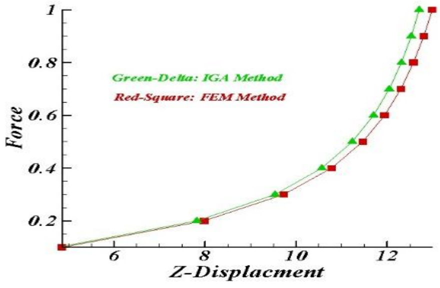


Fig. 3. Displacement in z direction of the end points

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Example 2.- Semi truncated cone

The aim of this example, further to showing the capability and performance of the method, is to study the effect of the number of integration Gauss quadrature points. The geometry, loading and boundary conditions are illustrated in Figure 4. This problem is solved by using three different cases where 27, 64, 216 Gauss integration points are used. The obtained results are depicted in Figure 5.

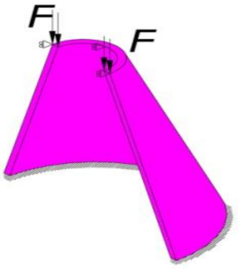
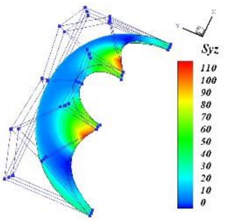
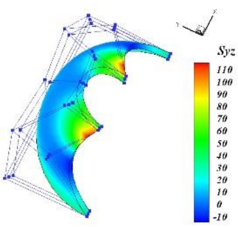


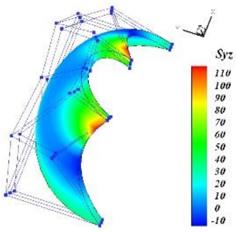
Fig. 4. Problem definition of semi truncated cone



(a)



(b)



(c)

Fig. 5. Contours of τ_{yz} for different, (a) 27, (b) 64, and (c) 216, Gauss integration points

Example 3- Bent plate

Besides demonstrating the performance of the method, the study of the number of load increments is the goal of this example. The geometry, loading and boundary conditions are illustrated in Figure 6. This problem is solved by using different numbers of load increments: 10, 20, 50, and 100. The obtained results are shown in Figure 7.

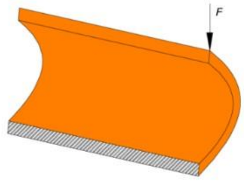


Fig. 6. Problem definition of bent plate

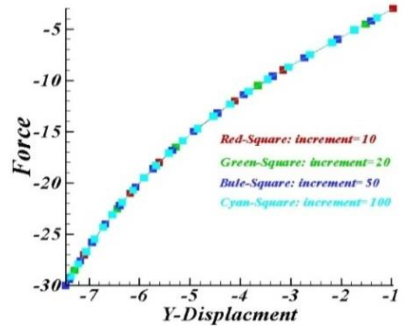


Fig. 7. Vertical displacement of the loaded point with different load increments

4. Conclusion

This paper aimed at the derivation of formulation and solution for nonlinear compressible elastic problems, known as compressible hyperelasticity, by the isogeometric analysis method. For this purpose, an algorithm was devised and by the linearization of governing equations the discretized equilibrium equations were obtained and the matrix of coefficients was derived by using the isogeometric concept. Then, the obtained results by the proposed method for compressible hyperelastic materials were compared with finite elements. Due to having large deformations in these kinds of problems, in the finite element method, apart from the need for remeshings in some problems, a relatively large number of elements are required that results in a large system of equations with high computational cost. In the isogeometric analysis method, by using the B-Spline and NURBS (Non-Uniform Rational B-Spline) basis functions that offer a good flexibility in geometrical modeling, need for remeshings is circumvented to a large extent. It is shown in this paper that in this approach, besides achieving good quality results, a smaller system of equations is obtained that reduces the computational cost. Also, the effects of the number of Gauss integration points, as well as the number of load increments, on the convergence of the solution are studied. The results indicate that for compressible hyperelastic materials, the final solution is independent of the manner in which the load increments are applied, and also the best choice of gauss integration point with quadratic basis functions is 64, four by four by four, Gauss points.