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Nonlinear Force Vibration of Functionally Graded Magneto-Electro-Elastic Rectangular Plate Based on the Third Order Shear Deformation Theory

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1. Introduction

In recent years, magneto-electro-elastic (MEE) materials have been the topic of many researches due to their ability to convert electrical, magnetic, and mechanical energy forms to each other. In this purpose, several researches around free and forced vibration of magneto-electro-elastic rectangular plates based on different theory of plates have been done. Also, the linear and nonlinear vibration had been considered for pure or layered MEE plates.

In this paper, the effects of several parameters on the nonlinear force vibration of a functionally graded magnetoelectro-elastic plate is investigated based on the Third Order Shear Deformation (TSDT) plate theory in conjunction with single-mode Galerkin and Lindeshtod-Poincare method.

2. Modelling the problem

Constitutive equations of a magneto-electro-elastic material are as follow:

$$C=\begin{bmatrix} C(z)_{11} & C(z)_{12} & 0 & 0 & 0\\ C(z)_{21} & C(z)_{22} & 0 & 0 & 0\\ 0 & 0 & C(z)_{44} & 0 & 0\\ 0 & 0 & 0 & C(z)_{55} & 0\\ 0 & 0 & 0 & 0 & C(z)_{66}\end{bmatrix}$$

$$e=\begin{bmatrix} 0 & 0 & e(z)_{31}\\ 0 & 0 & e(z)_{32}\\ 0 & e(z)_{24} & 0\\ e(z)_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, q=\begin{bmatrix} 0 & 0 & e(z)_{31}\\ 0 & 0 & e(z)_{32}\\ 0 & e(z)_{24} & 0\\ e(z)_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\epsilon=\begin{bmatrix} \epsilon(z)_{11} & 0 & 0\\ 0 & \epsilon(z)_{22} & 0\\ 0 & 0 & \epsilon(z)_{33}\end{bmatrix}, \mu=\begin{bmatrix} \mu(z)_{11} & 0 & 0\\ 0 & \mu(z)_{22} & 0\\ 0 & 0 & \mu(z)_{33}\end{bmatrix}$$

$$d=\begin{bmatrix} d(z)_{11} & 0 & 0\\ 0 & d(z)_{22} & 0\\ 0 & 0 & d(z)_{33}\end{bmatrix} \quad (1)$$

where C_{ij}, e_{31}, q_{31}, η_{33}, d_{33}, and μ_{33} are stiffness coefficient, piezoelectric, piezomagnetic, dielectric, magneto-electric, and magnetic permeability constants, respectively. σ_i, D_z, ψ, and B_z denote stress, electric potential, electric displacement along z-axis, magnetic potential, and magnetic flux density along z-axis, respectively. The plate is $CoFe_2O_4$-rich at z=+h/2 and $BaTiO_3$-rich at z=-h/2, and the material vary along the z-axis.

where *p* is a non-negative real number and *B* denotes the Corresponding author, piezoelectric phase.

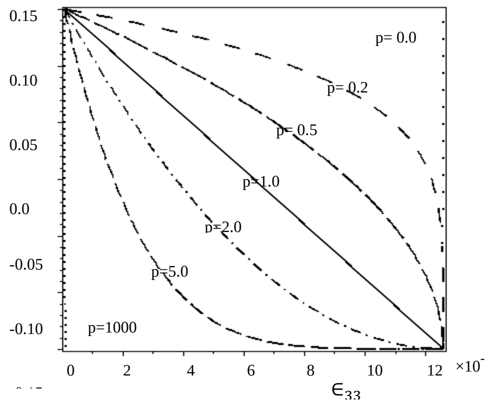

Fig. 1. Distribution of the volume fraction V_B during thickness for different values of the power law index P

Volume fraction of the piezoelectric phase (i.e., $BaTiO_3$) based on the power law is determined by:

$$V_B=\left(\frac{2z+h}{2h}\right)^{p} \text{ for } 0\le z\le +h \quad (2)$$

$$V_B+V_C=1 \quad (3)$$

$$P_{eff}=P_BV_B+P_CV_C \quad (4)$$

Equations of motion of a Functionally Graded Magnetoelectro Elastic Plate (FGMME) based on the third order shear deformation theory are expressed by:

$$N_{xx.x}+N_{xy.y}=I_0u_{0.tt}+(I_1-c_1I_3)\varphi_{x.tt}-c_1I_3w_{0.tt} \quad (5\text{-a})$$

$$N_{yy.y}+N_{xy.x}=I_0v_{0.tt}+(I_1-c_1I_3)\varphi_{y.tt}-c_1I_3w_{0.tt} \quad (5\text{-b})$$

$$\begin{aligned}&(N_{yy.y}+N_{xy.x})w_{0.y}+(N_{xx.x}+N_{xy.y})w_{0.x}+N_{yy}w_{0.yy}\\&+N_{xx}w_{0.xx}+c_1(P_{xx.xx}+P_{xy.xy}+P_{yy.yy})\\&+(Q_{x.x}-c_2R_{x.x})+(Q_{y.y}-c_2R_{y.y})+q_z\\&=I_0w_{0.tt}-c_1^2I_6(w_{0.ttxx}+w_{0.ttxx})\\&\quad+c_1I_3(u_{0.tt}+v_{0.tt})+c_1(I_4-c_1I_6)(\varphi_{x.xtt}+\varphi_{y.ytt})\end{aligned} \quad (5\text{-c})$$

$$\begin{aligned}&M_{xx.x}+M_{xy.y}-c_1(P_{xx.x}+P_{xy.y})+(Q_x-c_2R_x)\\&=-c_1(I_4-c_1I_6)w_{0.xtt}+(I_2-2c_1I_4+c_1^2I_6)\varphi_{x.tt}\\&+(I_1-c_1I_3)u_{0.tt}\end{aligned} \quad (5\text{-e})$$

$$\begin{aligned}&M_{yy.y}+M_{xy.x}-c_1(P_{yy.y}+P_{xy.x})+(Q_y-c_2R_y)\\&=-c_1(I_4-c_1I_6)w_{0.ytt}+(I_2-2c_1I_4+c_1^2I_6)\varphi_{y.tt}\\&+(I_1-c_1I_3)v_{0.tt}\end{aligned} \quad (5\text{-f})$$

Assuming the simply support boundary condition in all edge of the plate and using the following shape functions which satisfy the boundary condition ($\alpha=n\pi/a, \beta=m\pi/$):

and then substituting in Eq. (6) and finally applying the Galerkin method on the resulting equations, one can obtain equation 7:

It is seen that this equation includes quadratic and qubic nonlinearity terms, which in non-dimensional form can be written as equation 8:

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$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}(t)Cos\alpha xSin\beta y$$
$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}(t)Sin\alpha xCos\beta y$$
$$w_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}hW_{mn}(t)Sin\alpha xSin\beta y \tag{6}$$
$$\phi_x(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}(t)Cos\alpha xSin\beta y$$
$$\phi_y(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}(t)Sin\alpha xCos\beta y$$

$$Z_1W_{,tt}+Z_2W+Z_3WW_{,tt}+Z_4W^2+Z_5W^3=Z_6q_0\cos(\Omega t) \tag{7}$$

$$W_{,\tau\tau}+\omega_0^2W+\alpha_1WW_{,\tau\tau}+\alpha_2W^2+\alpha_3W^3=f\cos(\Lambda\Omega\tau) \tag{8}$$

3. Results and Discussion

To validate the proposed solution, the linear natural frequency of an MEE plate which is obtained from the present work has been compared with the related works and showed in Table (1-4).

Table 1. Comparison of normalized natural frequencies $\omega=\omega_0a\sqrt{\rho_{0_{max}}/C_{11_{max}}}$ of an isotropic square plate with effective elastic properties B Phase only

(*m*,*n*)				method
(3,1)	(2,2)	(1,2)	(1,1)	
7.160	6.226	4.559	2.397	present work
6.895	6.561	4.683	2.399	HSDT [7]

Table 2. Comparison of normalized natural frequencies $\omega=\omega_0a\sqrt{\rho_{0_{max}}/C_{11_{max}}}$ of an isotropic square plate with effective elastic properties C Phase only

(*m*,*n*)				method
(3,1)	(2,2)	(1,2)	(1,1)	
5.690	5.215	3.884	2.072	present work
6.009	5.196	3.834	2.065	HSDT [7]

Table 3. Comparison of normalized natural frequencies $\omega=\omega_0a\sqrt{\rho_{0_{max}}/C_{11_{max}}}$ of an isotropic Electro-elastic square plate

(*m*,*n*)				method
(3,1)	(2,2)	(1,2)	(1,1)	
7.178	6.235	4.568	2.323	present work
6.916	6.505	4.639	2.404	HSDT [7]

Table 4. Comparison of normalized natural frequencies $\omega=\omega_0a\sqrt{\rho_{0_{max}}/C_{11_{max}}}$ of an isotropic Magneto –elastic square plate

(*m*,*n*)				method
(3,1)	(2,2)	(1,2)	(1,1)	
5.974	5.220	3.892	2.760	present work
6.011	5.198	3.836	2.667	HSDT [7]

4. Nonlinear Forced Vibration Analysis

By using multiple time scales method and introducing detuning parameter σ as a quantity for deviation of excitation frequency Ω from linear frequency ω_0 , forced vibration analysis of the system in primary and Secondary resonance case have been investigated.

The stable state response in the primary resonance is presented as follow:

$$\left[\frac{9\alpha_3\omega_0^2-10\alpha_2^2+11\alpha_1\alpha_2\omega_0^2-\alpha_1^2\omega_0^4}{24\omega_0^2}r^2-\Lambda\sigma\right]^2r^2=\frac{Q^2}{4\omega_0^2} \tag{9}$$

Figure 2 shows the amplitude of the response as a function of amplitude of the excitation for several detuning parameter. For curvilinear values of the detuning parameter, the curves are initially tangent.

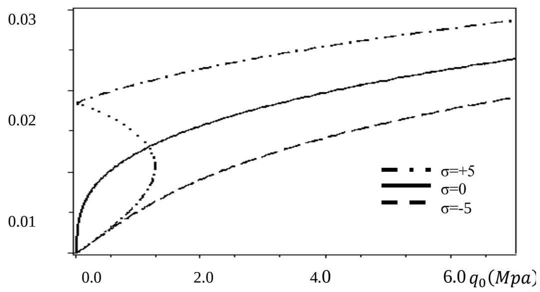

Fig. 2. Amplitude of the response as a function of amplitude of the excitation for several detuning

In Figure 3 the frequency-response curve is shown for two different excitation amplitude ranges. When the amplitude of the excitation increases, the response-frequency curve shifts from the axis σ=0 and goes upwards.

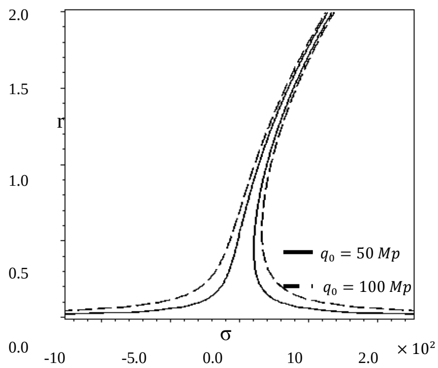

Fig. 3. Frequency-response curves for primary resonances- effect of amplitude of excitation

5. Conclusion

In this paper, nonlinear force vibrations of a functionally graded magneto-electro-elastic plate is investigated based on the third order shear deformation plate theory along with Lindeshtot-Poincare method and multiple time scales method. Several examples are given to validate the proposed solution and to investigate the effects of some parameters on the force vibration response of this plate.