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Two-dimensional transient dynamic analysis of decagonal quasicrystals subjected to shock loading using meshless generalized finite difference (GFD) method

S.M. Hosseini[1]

1. Introduction

In 1984, a new kind of crystals was discovered, which is called as quasicrystals. As compared with the usual crystals, the elastic behavior of QCs is different. In this kind of materials, there are two fields which are called phonon and phason fields. In quasicrystals, the phonon and phason fields are influenced on each other in static and dynamic loadings. So, the governing equations are in the coupled forms of PDEs in which the behaviors of both phonon and phason fields are related by a coupling parameter. To simulate the dynamic behaviors of QCs, the general dynamic theory for QCs should be used. To obtain the realistic dynamic behaviors of QCs, there are some theories for description of dynamic behaviors of QCs, which were presented in previous published papers. The elasto-/hydro-dynamic model for wave propagation and diffusion together with their interaction was employed for dynamic initiation of crack growth and crack fast propagation for the double cantilever-beam specimen (DCB) of two-dimensional decagonal Al–Ni–Co quasicrystals. Recently, an elastodynamic model of wave – telegraph type was proposed for the description of dynamic behaviors of QCs. The developed models for dynamic analysis of QCs such as elasto-hydrodynamics and wave type models give us the coupled set of PDEs. To solve the coupled PDEs, there are some analytical and computational (numerical) methods such as meshless methods and mesh based methods.

There is another numerical method based on the meshless approach called the generalized finite difference (GFD) method. The GFD method is a numerical method based on the meshless approach, which has a high capability for solving the coupled governing equations. In this paper, an effective meshless numerical method based on generalized finite difference (GFD) is employed for dynamic analysis of a two dimensional domain made of 2-D decagonal QCs, which is assumed to be under shock loading. The meshless approach is used for approximation of both the phonon and phason displacements and the stress tensors are approximated by using the constitutive relationships and derivatives of approximated displacements. The interaction between phonon and phason fields is studied in the presented dynamic analysis. Also, the effects of some parameters such as coupling parameter and the phason friction coefficient on dynamic behaviors of both phonon and phason fields are obtained for the problem.

2. Governing Equations

The equations of motion in the theory of compatible elastodynamics of quasicrystals in terms of the momenta and stresses take the form

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{1}$$

$$H_{ij,j} + g_i = D_{ij} \dot{w}_j \tag{2}$$

where f_i , g_i are the phonon body force and the phason body force, respectively, while D_{ij} is the phason coefficient of frictional force (dissipative force) vector. In isotropic case, $D_{ij} = D\delta_{ij}$ and $D > 0$. The generalized Hooke's law in two-dimensional elasticity for quasi-crystals are given as

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} + R_{ijkl}\omega_{kl} \tag{3}$$

$$H_{ij} = R_{klij}\varepsilon_{kl} + K_{ijkl}\omega_{kl} \tag{4}$$

where σ_{ij} and H_{ij} are the phonon and phason stresses, respectively, while the corresponding phonon strains ε_{kl} and phason strains ω_{kl} are defined in terms of gradients of the phonon displacements $u_i(\mathbf{x},t)$ and phason displacements $w_i(\mathbf{x},t)$ as

$$\varepsilon_{ij}(\mathbf{x},t) = \frac{1}{2}\left[u_{i,j}(\mathbf{x},t) + u_{j,i}(\mathbf{x},t)\right] \tag{5}$$

$$\omega_{ij}(\mathbf{x},t) = w_{i,j}(\mathbf{x},t) \tag{6}$$

It should be noted that ω_{kl} is not symmetric in contrast to ε_{kl} . The tensors of material coefficients and coupling coefficients are given as

$$\begin{aligned} c_{ijkl} &= c_{12}\delta_{ij}\delta_{kl} + c_{66}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\ c_{66} &= (c_{11} - c_{12})/2 \\ K_{ijkl} &= K_1\delta_{ik}\delta_{jl} + K_2\left(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}\right) \\ R_{ijkl} &= R\left[\left(\delta_{i1}\delta_{j2} + \delta_{i2}\delta_{j1}\right)\varepsilon_{3lk}\right] \\ &\quad + R\left[\left(\delta_{i1}\delta_{j1} - \delta_{i2}\delta_{j2}\right)\delta_{kl}\right] \end{aligned} \tag{7}$$

3. Numerical example and discussions:

To show some numerical results of the problem, a two dimensional domain is assumed with the following mechanical properties of Al-Ni-Co QCs. It is noted that the phonon displacement on one side of the assumed 2D domain is excited as suddenly increasing of phonon displacement (shock loading) to find the dynamic behaviors of both phonon and phason displacement fields. The influences of the variations in the phonon filed on the phason field are studied in this section. The transient behaviors of the phonon displacement based on the applied shock loading on one side of the 2D domain is illustrated by the spatial distributions at several time

[1] Professor, Industrial Engineering Department, Ferdowsi University of Mashhad, Mashhad, Iran.
sm_hosseini@um.ac.ir

instants. Additionally, the time evolution of phonon and phason fields at certain points of the analyzed domain are presented in the following discussions. Figure 1 shows the distribution of phonon displacement u_x along x direction at several time instants. The propagation of wave fronts in the field of phonon displacement u_x can be tracked in Fig. 1. It means that the phonon displacement wave front propagates with finite speed along x direction.

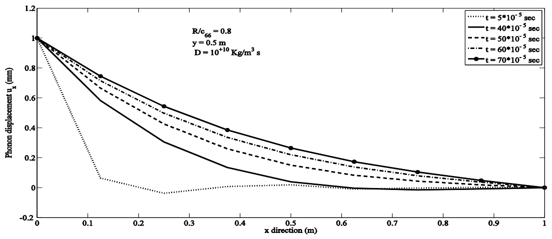

Fig.1. Variations of phonon displacement u_x along x direction at various time instants.

The transient behaviors of phonon and phason displacements can be found in Figs. 2.

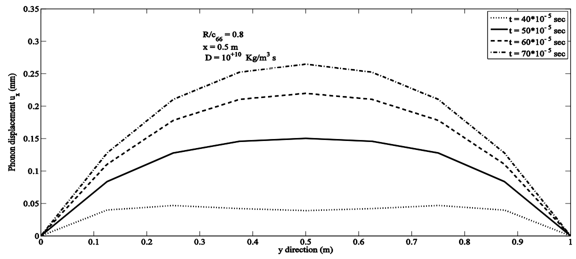

Fig. 2. Variations of phonon displacement u_x along y direction at various time instants.

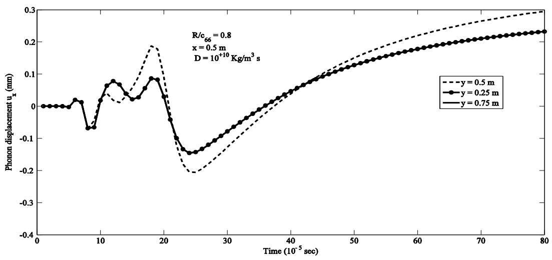

Fig. 3. Temporal variations of phonon displacement u_x at various positions.

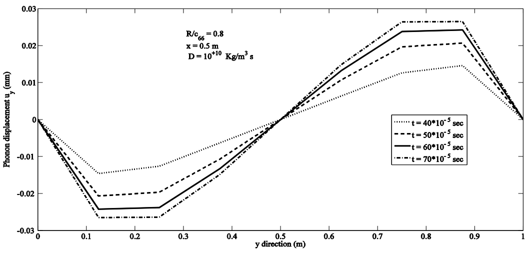

Fig. 4. Variations of phonon displacement u_y along y direction at various time instants.

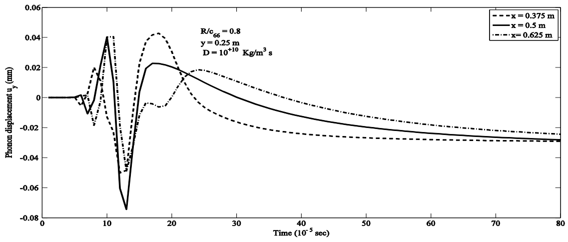

Fig. 5. Temporal variations of phonon displacement u_y at various positions.

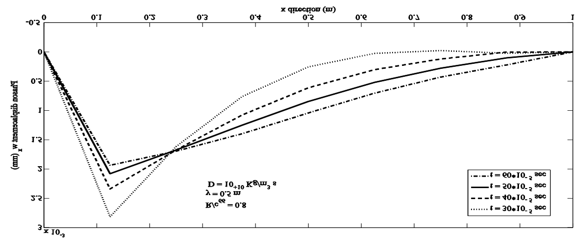

Fig. 6. Variations of phason displacement w_x along x direction at various time instants.

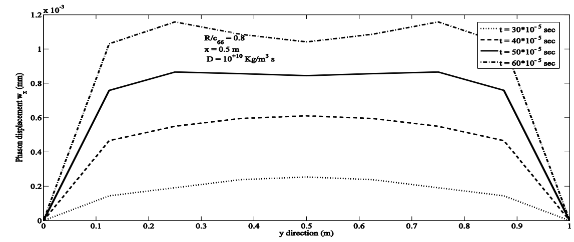

Fig. 7. Variations of phason displacement w_x along y direction at various time instants.

4.Conclusion

The elastodynamic analysis of two-dimensional decagonal Al-Ni-Co quasicrystals is carried out using meshless generalized finite difference (GFD) method in this article. The assumed two dimensional domains are excited by a shock loading applied on one side of the analyzed domain. The GFD method together with the Laplace transform technique with respect to time are employed for numerical solution of the present boundary value problem. To study on the dynamic behaviors of phonon and phason displacements fields, the phonon and phason field variables are transferred to time domain using the Laplace inversion technique based on the Talbot method. The wave propagations in both phonon and phason displacements fields are obtained and discussed in details. The effects of some parameters such as coupling parameter between phonon and phason fields on dynamic behaviors of fields' variables are studied. The wave fronts can be tracked at various times, which means that the wave fronts are propagated with finite speeds. The presented solution method, which is based on the GFD method has a high capability for the dynamic analysis of quasicrystals.