Dynamic Modeling and Optimal Walking Gait Planning of a Real Biped Robot Based on SLIP and Compass gait Models

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1- Introduction

Biped robots with point feet demonstrate faster gaits and more natural dynamics while their gait planning is very difficult due to their underactuation. This research focuses on modeling, optimization and gait generation of two different real biped models including a telescopic springy biped model and compass gait biped with kneeed swing leg. The main difference of these models with their corresponding theoretical models is that to give realization to the gait of these models the knee of their swing leg bends, clears the ground and straightens before touch-down. It increases degree of freedom and divides single support phase to two sub-phases. The main contribution of this research is to present real and practically implementable models for spring loaded inverted pendulum (SLIP) and compass gait theoretical models to compare their efficiency. To do so, the robot TARMER (fabricated in the University of Tabriz) is used for dynamic modeling and planning optimal gaits.

2- The robot model with telescopic springy leg

Walking dynamics of the telescopic springy biped model (Fig. 1) is based on SLIP model. The model has point feet, its torso angle is constrained and the robot moves in sagittal plane. Walking gait of this model includes single support sub-phases, touch-down event and double support phase that are modeled using Lagrange equations. Dynamic model of each continuous-time phase is derived as

\[
[D(q)]_{nxnx} \dot{q} + [C(q, \dot{q})]_{nxm} + [B]_{nxm} \alpha_{m} = 0
\]

in which, \( n \) is degree of freedom (DOF) and \( m \) is the number of control inputs.

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In single support sub-phase with kneeed swing leg \( n = 4 \) and \( m = 3 \), and the vector of control inputs including motors torques is \( \mathbf{u} = [u_{1}, u_{2}, u_{4}]^{T} \). The dynamic equations of the successive phases are combined together to make dynamic model of a walking gait. In single support sub-phase with straightened swing leg, the swing knee is locked and robot has 3 DOF with two control inputs as \( \mathbf{u} = [u_{3}, u_{5}]^{T} \) and in the dynamic equations \( n = 3 \) and \( m = 2 \).

In double support phase, both the feet are pined to the ground, the robot has 2 DOF and there is only one motor torque between legs. So in its dynamic model \( n = 2 \) and \( m = 1 \). Also touch-down model is derived using Lagrange impulse equation and two constraint equations of post touch-down velocities.

3- The robot model with kneeed compass leg

This model is similar to the previous model but its links are rigid without springs and its gait is based on compass gait model. The only difference is its kneeed swing leg to clear the ground and straightened again before touch-down to be compatible with real condition. In single support sub-phase with kneeed swing leg \( n = 3 \) and \( m = 3 \), and in single support sub-phase with straightened swing leg \( n = 2 \) and \( m = 2 \). Double support phase of this model takes place instantaneously within touch-down event.

4- Optimal walking gait planning

Optimal walking gait is derived by minimizing mechanical energy expenditure function named cost of
transport (COT) with the constraint of zeroing state vector error between two successive steps. Vector of optimization parameters includes initial condition of single support phase and discretized values of torques of motors during gait. Fitness function of the numerical optimization is assumes as the sum of energy expenditure function and penalty functions of constraints. Value of this function undergoes a descendent trend and converges to an optimal value after successive stages of optimization each having almost 100 iterations. Torques of the motors and stick diagram of the resulted optimal gaits have been shown in figures 2 to 5.

![Fig. 2. Optimal motor torques of the telescopic springy leg robot.](image)

![Fig. 3. Stick diagram of the optimal gait of the telescopic springy leg robot.](image)

![Fig. 4. Optimal motor torques of the kneed compass gait robot.](image)

![Fig. 5. Stick diagram of the optimal gait of the kneed compass gait robot.](image)

**5- Conclusion**

In this research, two practical biped mechanisms based on well-known theoretical models (SLIP and compass gait) were considered for a real robot and optimal walking gaits compatible with real condition were planned for them. However, the kneed telescopic springy biped model can reduce energy expenditure of the motors in running gaits, unexpectedly had a bit lower efficiency than the model without springs. This is because springs generating disorder in motion and wasting motors energy increase cost of transport. Furthermore, considering knee for swing leg and its flexion and deflexion in real condition increases COT. In the other model, however, the kneed compass gait model, existence of the knee for swing leg increases its COT relative to the theoretical compass gait model modeled by the other researchers.