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The Effect of Equilibrium Equation and Trefftz Functions on the Responses of Quadrilateral Bending Plate

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1- Introduction

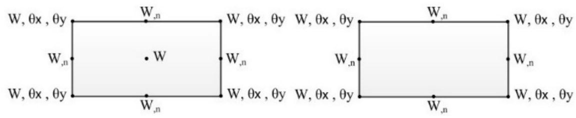
The finite element formulation of thin plate bending is based on the Kirchhoff's assumptions. In this theory, in contrast to Reissner-Mindlin's theory, the effects of shear deformation are neglected. Also, quadrilateral elements have a high efficiency in the modeling of bending plates due to their simple computations and acceptable accuracy. In conventional bending plate finite elements, increasing the number of degrees of freedom and raising the rank of interpolation polynomial lead to the more precise responses. This approach causes complex and time-consuming computations that are not so appealing to researchers. To overcome this issue, different methods have been employed so far. Among these attempts, finite element templates, hybrid and mixed formulations, such as, the hybrid-Trefftz finite element method have achieved a relative success.

In 1926, Trefftz suggested a new theory based on the variational principles and boundary integrations. In the T-type (Trefftz-type) finite element formulations, the internal fields are chosen so as to satisfy the governing differential equation analytically. In addition, interelement continuity and boundary conditions are enforced in an integral weighted residual way. Methods with more than one of the independent functions are called hybrid methods. In hybrid-Trefftz elements, two independent displacement functions are defined for internal field and boundaries. Another feature of hybrid-Trefftz elements is eliminating the rigid body motions, as it represents spurious zero energy modes and deficiency of the rank of the stiffness matrix.

In the present study, to improve the efficiency of the plate bending elements, the analytical solution of the governing differential equations is selected as the displacement interpolation functions. Depending on the type of loading, the analytical answer consists of general and particular solutions. The interpolation functions are derived from particular solutions, and Trefftz function is used as the answer of homogeneous part. Afterwards, due to the lack of boundary functions, the rigid body motions should be added to the Trefftz functions.

To achieve this aim, several groups of elements are studied and deficient versions are removed. The reason of eliminating these elements was the divergence of answers or singular stiffness matrix. Then, two groups of elements with two different arrangements of degrees of freedom are chosen. If the arrays of degrees of freedom and the arrangements of nodes are considered to be fixed, different functions can be employed in accordance to Trefftz function. Each function is derived by considering symmetry and removing some of the terms. Finally, four elements of the first group and six elements from the second group are selected. After conducting numerical tests, the results show that the completion of selected functions is not a necessary condition to increase the answers' accuracy. Comparing with complete functions, incomplete functions with fewer degrees of freedom, as long as they are properly selected, can produce more useful elements.

Since the interpolation functions meet the equilibrium conditions; forces are much more precise than displacements in many cases.



(a) (b)

Fig. 1. Arrangement of degrees of freedom for incomplete elements.

2- Finite element formulation

In the previous section, it was mentioned that the general analytical solution of governing fourth-order differential equation can be expressed as the form of Trefftz functions. T-complete solutions are a series of functions being complete in the sense of containing all possible solutions in a given solution domain which satisfy equilibrium equation. Therefore, each category of solutions has all terms with the order related to its category.

Subsequently, the finite element relations are formed by employing the minimum potential energy rules. The incomplete elements' shape functions are derived, based on the selected terms of Trefftz functions. Six elements are arranged similar to Fig. 1(b) with the name of ICTF-16-1 to 16-6 and four others (ICTF-17-1 to 17-4) have the Fig. 1(a) arrangement. Maintaining the symmetry of element's arrangement and type of degrees of freedom are the selection criteria. Previous researches have shown that $\partial w / \partial x, \partial w / \partial y, w$ and $w, \partial w / \partial n$ are suitable for the unknowns at the corners and sides, respectively.

Two complete elements (CTF-18-1, CTF-18-2) with fifth order shape function (fifth order T-complete function) are created for comparison. It should be noted that the arrangements of degrees of freedom of these elements are different.

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3- Numerical results

In this section, some numerical tests are presented to examine the suggested elements' efficiency. Different loading cases, such as, point loading, uniform distributed loading, sinusoidal and hydrostatic loading, and also different boundary conditions, such as, clamped edge, simply supported, free edge and column in corners either symmetric or asymmetric have been used. For each load case, the particular solution is not dependent on the boundary conditions. Furthermore, the verification for both displacement and force responses has been made. In the Figs. 2 and 3, the convergence rates of displacement and moment at the middle of a simply supported square plate under hydrostatic loading are shown. The results confirm the rapid convergence at a mesh of 16*16.

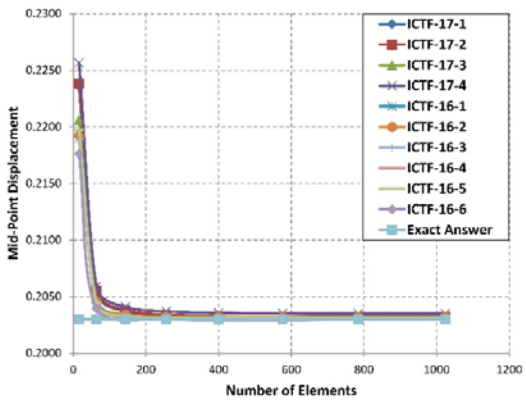

Fig. 2. Displacement convergence of a simply-supported plate under hydrostatic load.

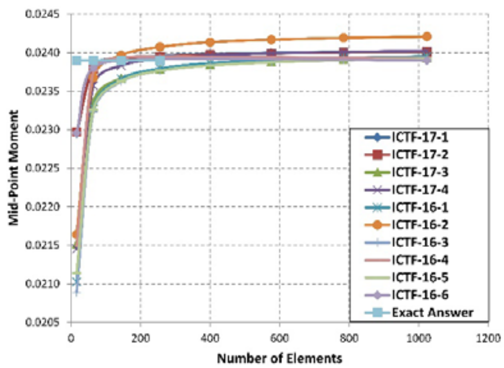

Fig. 3. Moment convergence of a simply-supported plate under hydrostatic load.

4- Conclusion

In this study, Trefftz functions was employed to model the lateral displacement of a Kirchhoff's bending plate. Shape functions were obtained by adding 1, x, y terms to the T-functions to ensure that the element represents the rigid body motions. Therefore, some elements with symmetric arrangements of degrees of freedom, based on the previous researches, were evaluated. After eliminating some of them due to the weak convergence of stiffness matrix instability, the top 10 elements were selected. It should be noted that some terms of Trefftz solutions are removed to represent the high-order element with fewer degrees of freedom.

In a T-complete function, each group of solutions is identified by the category number (k). Each solution of an even group (k=0, 2, 4,…) is symmetric with respect to x and y, but not in odd groups (k=1, 3,…). Therefore, choosing a right couple solution per odd group and one solution per even group may be appropriate. In addition, the solutions containing x^n and y^n will results in more stiff answers. Also, the effect of solutions with a low group number is negligible.

By adopting the displacement-based and force-based ranking process, elements ICTF-17-1, 17-2 and 17-4 gained the highest rating. The reason for their superiority is the presence of a middle node and a higher degree of freedom than ICTF-16 group elements. Thus, according to the rating of the ICTF-16 group's elements, the appropriate nodes' positioning and the number of degrees of freedom are more important than the choice of the shape functions. Comparing the answers of these elements with the complete elements' responses, it became evident that it was not necessary to choose all terms in the use of the Trefftz function. Hence, it is possible to use the incomplete function to raise the order of the shape function without increasing the number of degrees of freedom, and to obtain more precise answers.