Journal of Applied and Computational Sciences in Mechanics, Vol. 29, No. 2, 2018.

Dynamical Stability of Astrophysical Rings

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1- Introduction

In this paper the gravitational stability of a self-gravitating ring composed of n particles with mass m which are on a circular orbit around a central mass M is investigated. The first order perturbation analysis in the context of a modified theory of gravity known as MOG in the relevant literature is used. This theory has been introduced to address the dark matter problem. This problem is one of the serious challenges in the theoretical physics. More specifically astrophysical measurements show that there are evident deviations between Newton's gravitational theory and the observations. For example the rotation curves of spiral galaxies at large distances from galactic center are flat. While using the standard Newtonian gravity, these rotation curves should be decreasing function of radial distance.

There is also a serious mass discrepancy in the spherical galaxy clusters. In fact these clusters are almost six times more massive than what can be seen by telescopes. Consequently, physicists believe that there is a dark and unseen matter component in the universe which has not been detected yet. However although there are several advanced laboratories which use different techniques, the dark matter particles have not been observed yet.

Another approach to solve this enigma is the modified gravity. In other words, may be the current standard gravitational theory should be modified and there is no dark matter particle in the universe. MOG is a theory which follows the last picture. Mathematically this theory is more complicated than Einstein's general theory of gravity. In fact MOG in addition to the metric tensor possesses other classic fields which increase the degrees of freedom of the theory. More specifically MOG is a scalar-tensor-vector theory of gravity and includes two scalar field and a Proca vector field. These fields make MOG an effective theory to address the above mentioned dark matter problem.

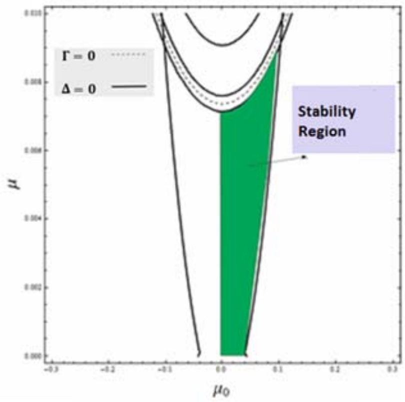

Fig. 1: The stability region for a n=7 system in MOG

2- Problem definition

This theory in the weak field limit where the gravity is not too strong and consequently the matter velocities are small compared to the speed of light, leads to a different gravitational potential. Therefore the gravitational force between point masses in this theory is different from the Newtonian case. More specifically the force in MOG is slightly stronger than the Newtonian gravitational force. For our ring system with radius r, MOG's circular velocity is larger and consequently the stability analysis of the system is, in principle, different from the standard case. On the other hand, the stability analysis in astrophysical systems leads to a better understanding of the underlying physical phenomena. For example using the stability analysis one can show that the Saturn's ring cannot be solid and should be consisted of small matter pieces. Moreover the stability analysis of galactic disks helps to investigate the origin and evolution of the spiral patterns. Using this approach astrophysicists have shown than spiral patterns are density waves which propagate on the surface of galactic disks as a small perturbation. Also stability issues help to understand the evolution and growth of the stellar bars in the center of disk galaxies. Moreover, several secular processes exist in the astrophysical systems which can be explained with the stability analysis.

As it already mentioned, in this paper the global stability of a ring around a massive spherically symmetric object, in the context of MOG is investigated. The final results may be extended to include MOG effects on the evolution of local stabilities in the real and large galactic rings. However, we restrict ourselves to a stellar system instead of a fluid ring.

In order to study the dynamical and global stability of the stellar ring system, first, the equilibrium state is derived. In other words, for a given radius of the ring the circular velocity and the positions of all point masses are found. The center of mass reference frame and complex coordinate system is used to reduce the mathematical difficulties. Then the first order perturbations to the position and velocities of the point masses is applied. By

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linearizing the governed differential equations, a dispersion relation for the propagation of the perturbations is found. As in any first order stability analysis, the dispersion relation is the most important result of the calculations.

3- Results and discussion

As usual one may use this relation in order to find the stability criterion. However, the dispersion relation is complicated and the stability criteria cannot be analytically derived. Therefore, a numeric procedure is used to find the criterion for two different cases:

a) Out-of-plane perturbations: In this case the particles only in the direction perpendicular to the plane of the ring are perturbed. It is shown that the ring is stable against these perturbations. In other words if the particles in this direction are perturbed, they will oscillate and the ring will retain its stability. It is also the case in Newtonian gravity. However, in principle the frequency of the oscillations will be different in these theories. More specifically one may expect that perturbations oscillates more rapidly in MOG than in Newtonian case.

b) In-plane perturbations: In this case the particles in the azimuthal and radial directions are perturbed. Since the analysis cannot be done for arbitrary number of particles n, two different cases of n=7 and n=8 are studied. The results have been shown in Figs. 1 and 2. It is necessary to mention that MOG possesses two free parameters α and μ_0 which their observational value have been measured using rotation curves data of spiral galaxies. It is assumed that $\alpha = 8.89$ and different values of μ_0 are investigated. On the other hand, the parameter μ is the ratio of a point mass in the ring to the central mass.

As it is clear in Fig. 1, when $\mu_0 = 0$ the Newtonian stability criterion, i.e. $\mu \leq 0.00715$, is recovered. On the other hand by increasing the magnitude of μ the stability criterion is changed and more massive particles get also stable against these perturbations. In other words, MOG makes the ring more stable against all kind of small perturbations. However, as it is shown in the Fig. 1, the stability region does not cover large values of μ. In fact when this parameter is larger than 0.1 the gravitational force grows rapidly and makes the ring unstable. The current value of μ is small enough to lay in the stability region.

A similar behavior for an eight-particle system is found. The stability region has been illustrated in Fig. 2. In this case also one may recover the Newtonian stability criterion by setting the free parameter to zero. The stability region is wider and include larger values for μ_0 and μ.

For a more real analysis one may study a fluid ring system. In this case one may linearize the Euler's equation, continuity equation and the modified Poisson equation in MOG; and investigate the local and global stability of the ring. Such a study would help to interpret the star formation rates in the galactic rings.

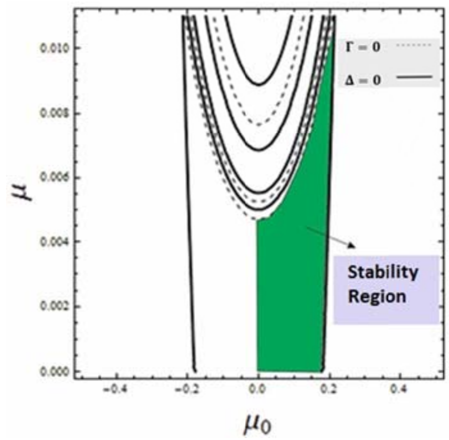

Fig. 2: The stability region for a n=8 system in MOG.