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Estimating Simultaneously Location and Intensity of Heat Source in a Functionally Graded Plate using Inverse Solution

M. Shamsa[1], M.R. Golbahar Haghighi[2],
P. Malekzadeh[3]

1- Introduction

In some problems, the direct measurement of some parameters is experimentally impossible because of two reasons. Firstly, in some cases the temperature of internal surfaces is very high and sensors are not able to measure them and sustain their integrity. Secondly, the measurement equipment are too complicated and expensive. Today the inverse methods are used to estimate some thermal parameters.

The problem under investigation in this paper is the estimation of intensity and location of a point heat source by using the measured temperature at some known points on the boundary. The necessity of employing this approach arises from the impossibility of direct measuring of the location and intensity of point heat source in sometimes.

2. Mathematical modeling

We have a two-dimensional heat conduction problem of a thin FG plate under an unknown time varying intensity and location point heat source (Fig. 1). The heat capacity, thermal heat conductivity and density are assumed to be known. In continuation, the solution procedures of problem are explained.

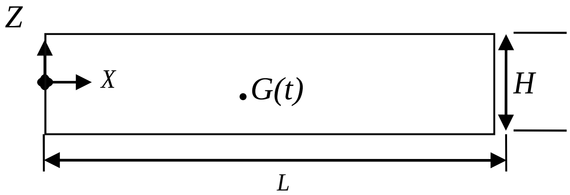

Fig. 1 Geometry of plate.

3. Solution stages of inverse problem

- The conjugate gradient method together with adjoint problem are employed to estimate the intensity of the heat source.
- The conjugate gradient method is applied to determine coordinate (x_0, z_0) of the heat source.

The new coordinate and the conjugate gradient method is employed to estimate the new value of heat source intensity. This iterative procedure is continued until the convergence criteria are satisfied.

4. Direct problem

The governing equation, boundary and initial conditions for the transient heat transfer problem are, respectively [22].

$$k\frac{\partial^2 T}{\partial x^2}+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+G\delta\left(x-x_0\right)\delta\left(z-z_0\right)=\rho c\frac{\partial T}{\partial t} \tag{1}$$

$$\frac{\partial T}{\partial x}=0 \quad \text{on } x=0 \tag{2.a}$$

$$\frac{\partial T}{\partial z}=0 \quad \text{on } z=0 \tag{2.b}$$

$$T=T_i \quad \text{on } z=H \tag{2.c}$$

$$T(x,z,0)=T_0(x,z) \tag{2.d}$$

Equation (1) is solved using the finite element method.

5. Numerical results

In the first example the following heat source at the center of the plate is considered

$$G(t)=\begin{cases} 2G_0\left(\dfrac{t}{t_f}\right) & 0\le \dfrac{t}{t_f}\le 0.5 \\ 2G_0\left(1-\dfrac{t}{t_f}\right) & 0.5\le \dfrac{t}{t_f}\le 1 \end{cases} \tag{3}$$

In Fig. 2. the estimated intensity of heat source of the first example without considering the measurement errors is shown.

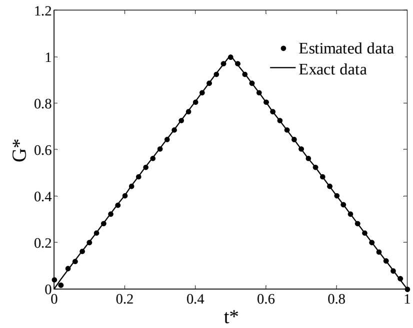

Fig. 2 Comparison of the exact and estimated value of the strength for the point heat source in example 1($\sigma = 0$).

[1] Master of Science, Department of Mechanical Engineering, Persian Gulf University, Bushehr, Iarn.
[2] Associate Professor, Department of Mechanical Engineering, Persian Gulf University, Bushehr, Iarn.
Email: golbahar@pgu.ac.ir
[3] Professor, Department of Mechanical Engineering, Persian Gulf University, Bushehr, Iarn.

In the second example, the following heat source at the center of plate is considered

$$G(t)=\begin{cases} 3G_0\left(\dfrac{t}{t_f}\right) & 0\le\dfrac{t}{t_f}\le\dfrac{1}{3} \\ G_0 & \dfrac{1}{3}\le\dfrac{t}{t_f}\le\dfrac{2}{3} \\ 3G_0\left(1-\dfrac{t}{t_f}\right) & \dfrac{2}{3}\le\dfrac{t}{t_f}\le 1 \end{cases} \tag{4}$$

The estimated intensity of heat source in the second example is drawn in Fig. 3. Also, the location of the heat source is estimated as

$$x_0(\text{exact})=0.4865,\ z_0(\text{exact})=0.5242 \tag{5}$$

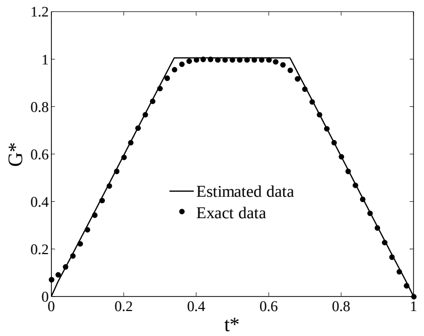

Fig. 3 Comparison of the exact and estimated value of strength of the distributed heat source in example 4 for $(\boldsymbol{\sigma}=\mathbf{0})$.

6. Conclusion

In this paper, the conjugate gradient algorithm was employed to estimate the location and intensity of the point heat source. It was found that the rate of convergence and accuracy of the method for this problem is very good and it is a stable method. Different functions were considered for the heat source and it was observed that the computation algorithm can estimate them.