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Similarity Solution of Axisymmetric Stagnation Point Flow of Nanofluid on Rotating Cylinder

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1- Introduction

Nanofluid is a name first applied by Choi, referring to fluids containing solid suspended nanoparticles smaller than 100 nm with volume fraction of less than 5 percent. It can enhance heat transfer performance compared to pure liquids. Nanofluids can be used to improve thermal management system in many engineering applications such as heat transfer, micromechanics and instrument, HVAC system and cooling devices.

In this research, similarity solution of axisymmetric stagnation-point flow and heat transfer of Nanofluid impinging on rotating cylinder with uniform angular velocity has been presented for the first time. Flow is considered in cylindrical coordinates (r,φ,z) with corresponding velocity components (u,v,w) as Fig. 1.

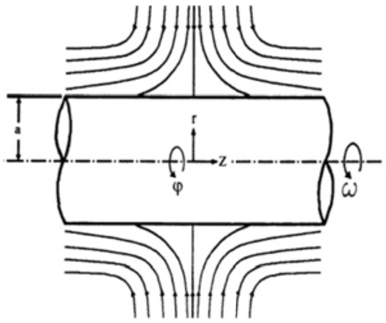

Fig. 1. Schematic of stagnation point flow on rotating cylinder.

2- Properties of Nanofluid

The aluminum oxide (γAl_2O_3) nanoparticles used in this research have the following characteristics:

Density ρ_m = 3,880 kg/m^3, mean particle diameter is 44 nm.

2-1 Nanofluid density

It is assumed that the density of the aluminum oxide nanoparticles is constant in the entire range of considered temperature. The following relation is used to calculate the nanofluid density:

$$\rho_n = (1-\phi_v)\rho_f + \phi_v \rho_p \qquad (1)$$

where subscripts n, f and p denote the nanofluid, base fluid and the particles respectively, and ϕ_v is the particle fraction.

2-2 Dynamic viscosity of Nanofluid

The viscosity of the nanofluid can be estimated with the existing relations for the two phase mixture. In 2011, Massimo Corcione proposed empirical correlation for predicting the relative viscosity,

$$\frac{\mu_n}{\mu} = \frac{1}{1-34.87(\frac{d_p}{d_f})^{-0.3}\phi_v^{1.03}} \qquad (2)$$

where d_f is the equivalent diameter of a base fluid molecule, given by:

$$d_f = 0.1(\frac{6M}{N\pi\rho_{f0}})^{\frac{1}{3}} \qquad (3)$$

in which, M is the molecular weight of the base liquid, N is the Avogadro number, and ρ_{f0} is the mass density of the base liquid calculated at temperature $T_0 = 293k$.

2-3 Thermal conductivity coefficient of Nanofluid

The following relation is used to calculate thermal conductivity of Nanofluid:

$$\frac{k_{eff}}{k_f} = 1 + 4.4\,\mathrm{Re}_p^{\ 0.4}\,\mathrm{Pr}_{bf}^{\ 0.66}(\frac{T}{T_{fr}})^{10}(\frac{k_p}{k_f})^{0.03}\phi_v^{\ 0.66} \qquad (4)$$

where $\mathrm{Re}_p = \frac{2\rho_{bf}k_bT}{\pi\mu_{bf}^{\ 2}d_p}$ & $\mathrm{Pr}_{bf} = \frac{\mu_{bf}(c_p)_{bf}}{k_{bf}}$

3- Governing equations

A reduction of the Navier-Stokes equations is obtained by the following coordinate separation of the velocity field and by introducing $\eta = (\frac{r}{a})^2 - 1$, we have:

$$u = -\bar{k}\frac{a}{\sqrt{\eta+1}}f(\eta)\ ,\ v = \bar{k}\frac{a}{\sqrt{\eta+1}}G(\eta),$$
$$w = 2\bar{k}\,f'(\eta)\,z,\quad P = \rho_n\bar{k}^2a^2p \qquad (5)$$

Transformations (5) yields an ordinary differential equation system in terms of f and G as

$$(\eta+1)f''' + f'' + \mathrm{Re}_n[1-(f')^2 + f\,f''] = 0$$
$$(\eta+1)G'' + \mathrm{Re}_n\ f\,G' = 0 \qquad (6)$$

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In these equations,

$$\mathrm{Re}_n = \beta \frac{\bar{k}\, a^2}{2\upsilon_f} \quad (7)$$

$$\beta = [1 - 34.87(\frac{d_p}{d_f})^{-0.3} \phi_v^{\,1.03}](1 - \phi_v + \phi_v \frac{\rho_p}{\rho_f}) \quad (8)$$

To transform the energy equation into a non-dimensional form, we introduce dimensionless temperature θ when the wall temperature or wall heat flux is constant

$$\theta(\eta) = \frac{T(\eta) - T_\infty}{T_w - T_\infty}:$$ when the wall temperature is constant

$$\theta(\eta) = \frac{T(\eta) - T_\infty}{\frac{aq_w}{2k_{bf}}}$$: when the wall heat flux is constant

By using Corcione's correlation and introducing Γ as

$$\Gamma = 4.4(\frac{2\rho_{bf} K_b}{\pi \mu_{bf}^{\ \ 2} d_p})^{0.4} \frac{1}{T_{fr}^{\ 10}} (\frac{k_p}{k_f})^{0.03}$$

The final form of energy equation for constant wall temperature and constant wall heat flux is introduced by Eqs. 9 and 10, respectively.

$$\begin{aligned} &\left\{1 + \Gamma \phi_v^{\,0.66}\, \mathrm{Pr}_{bf}^{\,0.66} [T_\infty + (T_w - T_\infty)\theta]^{10.4}\right\} \\ &[(\eta+1)\theta'' + \theta'] + 10.4[T_\infty + (T_w - T_\infty)\theta]^{9.4} \\ &(T_w - T_\infty)\Gamma \phi_v^{\,0.66}\, \mathrm{Pr}_{bf}^{\,0.66} (\eta+1)(\theta')^2 \\ &+ \left\{1 - \phi_v + \phi_v [\frac{(\rho c_p)_p}{(\rho c_p)_f}]\right\} \mathrm{Re}_{bf}\, \mathrm{Pr}_{bf}\, f\theta' = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} &\left\{1 + \Gamma \phi_v^{\,0.66}\, \mathrm{Pr}_{bf}^{\,0.66} [T_\infty + \frac{aq_w}{2k_{bf}}\theta]^{10.4}\right\} \\ &[(\eta+1)\theta'' + \theta'] + 10.4[T_\infty + \frac{aq_w}{2k_{bf}}\theta]^{9.4} \\ &\frac{aq_w}{2k_{bf}} \Gamma \phi_v^{\,0.66}\, \mathrm{Pr}_{bf}^{\,0.66} (\eta+1)(\theta')^2 + \\ &\left\{1 - \phi_v + \phi_v [\frac{(\rho c_p)_p}{(\rho c_p)_f}]\right\} \mathrm{Re}_{bf}\, \mathrm{Pr}_{bf}\, f\theta' = 0 \end{aligned} \quad (10)$$

4- Results

The effect of variations of particle fraction factor on the dimensionless function $\frac{2\upsilon \mathrm{Re}}{a^2\omega} G(\eta)$ against η for Re=10 is depicted in Fig. 2. As the particle fraction increases, the value of the dimensionless function $\frac{2\upsilon \mathrm{Re}}{a^2\omega} G(\eta)$ increases and increasing the value of this function means increasing the angular component of the fluid velocity field.

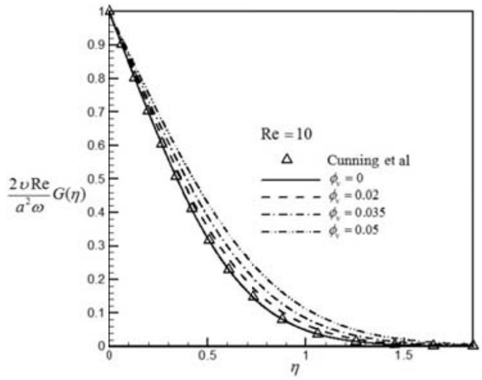

Fig. 2. Effect of particle fraction on angular component of the fluid velocity field.

Effect of variations of particle fraction factor on $\theta(\eta)$ function against η for $T_w = 450K$ and Re=0.1 is presented in Fig. 3. For $\phi_v = 0$, base fluid, the result of Gorla is extracted; it is interesting to note that, as ϕ_v increases, the absolute value of the dimensionless temperature gradient is decreased at surfaces; nevertheless, this decreasing rate is negligible compared to that of increasing rate in the thermal conductivity. Therefore, the heat transfer coefficient is increased through addition of nanoparticles.

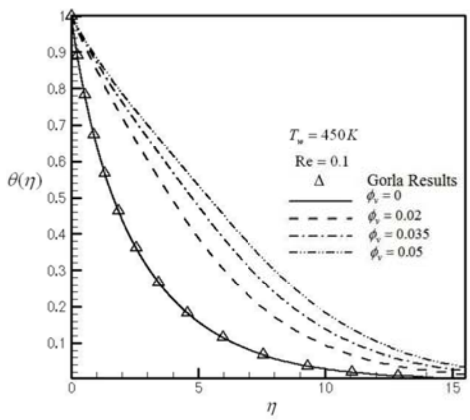

Fig. 3. Effect of particle fraction on dimensionless temperature.

5- Conclusion

In the present study, similarity solution of axisymmetric stagnation point flow and heat transfer of Nanofluid on rotating cylinder with constant angular velocity is presented. Results show that:

- Increasing the particle fraction would decrease radial component of velocity field and axial component of wall shear stress.
- Increasing the particle fraction would increase heat transfer coefficient, angular component of velocity field and angular component of wall shear stress.