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Boundary Condition Identification of a Clamped Beam in Flexible Support

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1-Introduction

Boundary conditions play a key role in dynamic response and modal characteristics of mechanical structures. Since modeling and dynamic analysis of real structures containing real boundary conditions and joints is a difficult task to do in structural dynamics, it is important to have accurate dynamic models for boundary conditions and joints. Many methods proposed for identification of joints or boundary condition parameters in the past. It is worth mentioning that having an accurate physical understanding of the structure is vital in successfully using the identification methods.

Identification of joints and boundary conditions parameters has been studied and many methods have been proposed including the model updating based methods. In this paper identification of stiffness parameters of boundary conditions are considered using simulation and experimental results. Dynamic modeling of a beam containing elastic boundary condition is constructed in order to extract its characteristic equation. An identification method for boundary condition parameters is proposed using the characteristic equation. First, the accuracy of the proposed method is verified using simulation results. The natural frequencies and estimations for stiffness parameters of boundary conditions are obtained using a FE model. Using the natural frequencies in the proposed method the boundary condition stiffness parameters are identified and are compared with the estimated values. Next, experimental results are used for identification of the boundary condition parameters. In the next section the problem statement is described.

2- Problem statement

A beam-like structure with rectangular cross section which is fixed by a flexible support in one end and free at the other end is considered (Fig 1a). The stiffness parameters of flexible support change modal characteristics like natural frequencies and mode shapes of the beam. Therefore, using of experimental or simulation modal characteristics and employing an identification approach the stiffness parameters of the support can be identified. In mathematical modeling shown in Fig 1b the beam section is considered using Euler-Bernoulli beam theory and the elastic support is represented by two lateral and torsional springs which are the equivalent stiffness of the elastic support. Their stiffness coefficients affect the dynamic characteristics and natural frequencies of the beam.

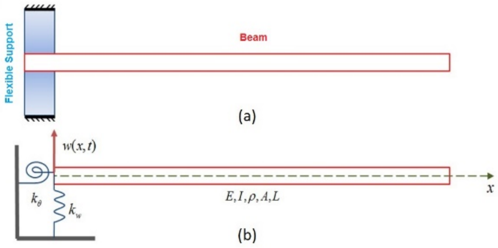

Fig. 1. (a) Beam with elastic support, (b) its mathematical representation

3-Dynamic Modeling

The dynamic modeling of the structure which will be used in natural frequency calculation is presented in this section. The equation governing free vibration of the beam is,

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = 0 \quad (1)$$

where $w(x,t)$, E, I, ρ and A are respectively lateral displacement, modulus of elasticity, cross sectional moment of inertia, density and cross sectional area. The boundary conditions are,

$$EI\frac{\partial^3 w(x,t)}{\partial x^3}\bigg|_{x=0} + k_w w(x,t)|_{x=0} = 0 \quad (2)$$

$$EI\frac{\partial^2 w(x,t)}{\partial x^2}\bigg|_{x=0} - k_\theta \frac{\partial w(x,t)}{\partial x}\bigg|_{x=0} = 0 \quad (3)$$

$$EI\frac{\partial^2 w(x,t)}{\partial x^2}\bigg|_{x=L} = 0 \quad (4)$$

$$EI\frac{\partial^3 w(x,t)}{\partial x^3}\bigg|_{x=L} = 0 \quad (5)$$

k_w and k_w are stiffness of the boundary condition. By considering the free vibration response as $w(x,t) = Y(x)\sin(\omega t)$ – $Y(x)$ and ω are respectively mode shape and natural frequency- and substituting it in equations (1-5) one obtains,

$$Y''''(x) - \lambda^4 Y(x) = 0, \quad \lambda^4 = \rho A\omega^2/EI \quad (6)$$

$$EIY'''(0) + k_w Y(0) = 0 \quad (7)$$

$$EIY''(0) - k_\theta Y'(0) = 0 \quad (8)$$

$$EIY''(L) = 0 \quad (9)$$

$$EIY'''(L) = 0 \quad (10)$$

By solving equation (6) the mode shapes are obtained as,

$$Y(x) = C_1\sin(\lambda x) + C_2\cos(\lambda x) + C_3\sinh(\lambda x) + C_4\cosh(\lambda x) \quad (11)$$

The mode shape defined in equation (11) has to satisfy equations (7-10). By substituting equation (11) into equations (7-10) a set of algebraic equation is obtained as follows.

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$$[D(\omega)]\{c\} = \begin{bmatrix} -\lambda^3 & k_1 & \lambda^3 & k_1 \\ -k_2 & -\lambda & -k_2 & \lambda \\ -\cos(\lambda L) & \sin(\lambda L) & \cosh(\lambda L) & \sinh(\lambda L) \\ -\sin(\lambda L) & -\cos(\lambda L) & \sinh(\lambda L) & \cosh(\lambda L) \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (12)$$

where $k_1 = \frac{k_w}{EI}$ and $k_2 = \frac{k_\theta}{EI}$. By solving characteristic equation, i.e. $|D(\omega)| = 0$, natural frequencies are obtained. In the following an identification approach is proposed using $[D(\omega)]$. First one can write $[D(\omega)]$ as,

$$D(\omega) = \overline{D}\,(\omega) + K_J \quad (13)$$

K_J may be expressed as $K_J = k_1 u_1 v_1^T + k_2 u_2 v_2^T$ where,

$$[u_1 \quad v_1 \quad u_2 \quad v_2] = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (14)$$

By substituting natural frequencies $\omega_j, j = 1,2,\dots,n$ in equation (13) we have,

$$D(\omega_j) = \widehat{D}_s(\omega_j) + k_s u_s v_s^T\,, \quad s = 1 \text{ or } 2 \quad (15)$$

Based on characteristic equation $|D(\omega_j)| = 0$ which will results in the following equation in term of unknown parameters of boundary condition,

$$P_j(k_1,k_2) = P_j(\{k\}) = |\widehat{D}_s(\omega_j)|\left(1 + k_s v_s^T \widehat{D}(\omega_j)^{-1} u_s\right) = 0$$
$$j = 1,2,\dots,n\ ,\ s = 1 \text{ or } 2 \quad (16)$$

The boundary condition parameters can be identified by solving above equation.

4-Identification Using Simulated Results

A beam structure having rectangular cross section is modelled and analyzed in *ANSYS* in order to obtain its natural frequencies. By substituting natural frequencies in equation (16) and solving it iteratively, the parameters of the boundary condition are identified. The variation of objective function and support parameters during identification are shown in figure 2. The identified parameters are $k_w = 5.829 \times 10^5$ N/m and $k_\theta = 175.229$ Nm/rad. The natural frequencies are compared in table 1.

5-Identification Using Experimental Results

In this section a steel beam as shown in figure 3 is considered. The natural frequencies are measured by doing experimental modal analysis. A measured FRF is shown in figure 4. Using the natural frequencies in equation (16) the boundary condition parameters are identified as $k_w = 7.983 \times 10^6$ N/m and $k_\theta = 4060.7$ Nm/rad. Figure 2 shows the objective function and support parameters during identification. In table 1 the experimental and identified natural frequencies are compared.

6- Conclusions

In this paper boundary condition parameter identification of elastically supported beams were considered. An identification approach was proposed based on using the natural frequencies in characteristic equation. The applicability of the proposed method was verified using simulated and experimental results which indicates acceptable accuracy in identification.

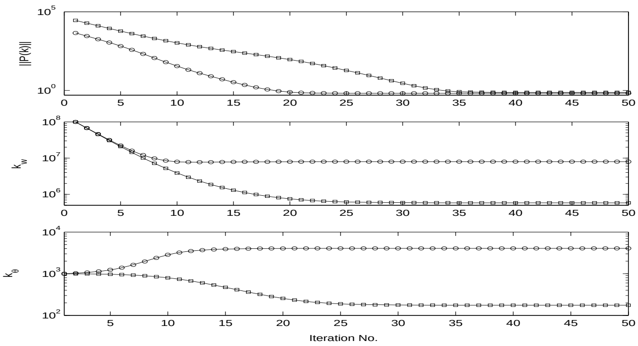

Fig. 2. Variation of objective function and support parameters, simulated (□), experiment (o)

Table 1. Comparison of identified results with simulated and measured natural frequencies

	1st mode	2nd mode	3rd mode	4th mode
Simulated	24.41	227.80	694.73	1384.9
Identified	24.40	230.43	670.24	1234.8
Error (%)	-0.02	1.16	-3.46	-10.41
Measured	52	329	924	1832
Identified	52	331.29	913.27	171.04
Error (%)	-0.01	0.69	-1.16	-6.64

Fig. 3. Modal test set up

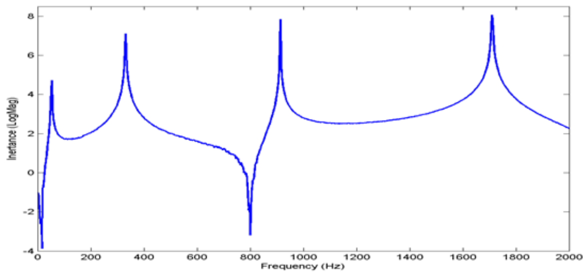

Fig. 4. Measured FRF