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Evaluation of Crack Growth Criteria in Functionally Graded Polymer using Finite Element Method

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1-Introduction

Functionally graded materials (FGMs) are composites that their properties change as a continuous function of position. For the first time in 1984, the Japan aerospace laboratory employed the materials which were produced with heterogeneous microstructure and their mechanical properties gradually and continuously varied from one surface to another. Due to the continuity in change, residual stresses and the stress intensity factors (SIFs) are reduced. FGMs include wide range of commercial applications such as tools, biomedical devices, optical fibers and coatings resistant to abrasion and heat. Researches reveal that the most common failure mode in these materials is crack initiation and its growth.

2- Fracture Criteria

Stress field near crack is considered to be linear elastic in FGMs and can be expressed as the following relations:

$$\sigma_{rr} = \frac{1}{\sqrt{2\pi r}} cos\frac{\theta}{2} [K_I \left(1 + sin^2\frac{\theta}{2}\right) + \frac{3}{2} K_{II} \left(sin\,\theta - 2\,tan\frac{\theta}{2}\right)] \tag{1}$$

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} cos\frac{\theta}{2} [K_I\,cos^2\frac{\theta}{2} - \frac{3}{2} K_{II}\,sin\,\theta] \tag{2}$$

$$\sigma_{r\theta} = \frac{1}{2\sqrt{2\pi r}} cos\frac{\theta}{2} [K_I\,sin\,\theta + K_{II}(3\cos\theta - 1)] \tag{3}$$

Important note about the stress fields in the FGMS is that they behave like a homogeneous material since the field is studied near the crack tip and due to the small size of the study area. The local homogeneity makes possible to use homogeneous fracture criteria for the FGMs with the difference that the conditions are investigated near the crack tip.

2-1-Maximum Tangential Stress Criterion

According to this criteria, crack will propagate in direction of the maximum tangential stress ($\sigma_{\theta\theta}$) and in this direction, shear stress ($\sigma_{r\theta}$) is zero. The crack initiation angle is achieved through the following equation:

$$\frac{\partial\sigma_{\theta\theta}}{\partial\theta} = 0\,, \frac{\partial^2\sigma_{\theta\theta}}{\partial\theta^2} < 0 \;\rightarrow\; \theta = \theta_0 \tag{4}$$

2-2 Maximum Strain Energy Release Rate

In accordance with this criterion, crack will propagate in the direction of maximum energy release rate and it will begin to grow when the maximum energy release rate reaches a critical value:

$$\frac{\partial\mathcal{G}(\theta)}{\partial\theta} = 0\,, \frac{\partial^2\mathcal{G}(\theta)}{\partial\theta^2} < 0 \;\rightarrow\; \theta = \theta_0 \tag{5}$$

2-3 Minimum Strain Energy Density

This criterion predicts that crack grows in the direction which the strain energy density factor S is minimized. Therefore, crack initiation angle, θ_0 is determined as follows:

$$\frac{\partial S(\theta)}{\partial\theta} = 0\,, \frac{\partial^2 S(\theta)}{\partial\theta^2} > 0 \;\rightarrow\; \theta = \theta_0 \tag{6}$$

3- Interaction Integral Method

Interaction integral method is an accurate method to calculate SIFs in the FGMs. This method applies auxiliary field such as displacement (u^{aux}), strain (ε^{aux}) and stress (σ^{aux}) field to estimate SIFs in mixed mode. Interaction integral is obtained from the J-independent integral, provided by Rice for cracked FGMs in the elastic and both real and auxiliary condition as follows:

$$M = \int_A \left\{\sigma_{ij}u_{i,1}^{aux} + \sigma_{ij}^{aux}u_{i,1} - \sigma_{ik}\varepsilon_{ik}^{aux}\delta_{1j}\right\} \cdot q_{,j}dA + \int_A \left\{\sigma_{ij}^{aux}u_{i,1} - C_{ijkl,1}\varepsilon_{kl}\epsilon_{ij}^{aux}\right\} q dA \tag{7}$$

4-Finite Element Simulation

Fracture analysis software FRANC2D is used to simulate crack initiation and propagation. The material property variations are defined as linear functions such as elastic modulus, Poisson's ratio, fracture toughness, thickness (if needed), etc. After finite element simulation, achieved numerical results are compared with the experimental results of Abanto and Lambros. It should be noted that the stresses in the un-cracked plate are numerically determined and the accuracy of FGM behavior is validated.

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4-1 Oblique Crack and Varying Properties in the Lateral Direction

Variation of the elastic modulus, fracture stress and strain, geometric dimensions and crack orientation are shown in Figure 1.

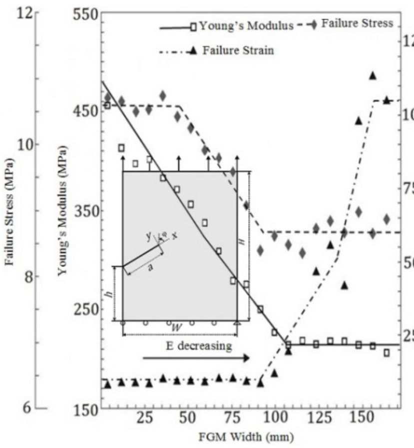

Fig. 1. Geometric dimensions of FGM plate with oblique crack and variation of properties in the lateral direction

The resulting of SIFs and crack initiation angle is compared with the experimental results in Table 1.

Table 1 Comparison of SIFs and crack initiation with the experimental results for FGM plate with oblique crack and variation of properties in the lateral direction

Result	$K_I(MPa\sqrt{m})$	$K_{II}(MPa\sqrt{m})$	Initiation angle	Error (%)
Exp.	0.755	0.179	-28 ± 1.5	
$\sigma_{\theta\theta_{Max}}$	0.735	0.225	-29.53	5.46
$\mathcal{G}(\theta)_{Max}$	0.735	0.225	16.61	159.32
$S(\theta)_{Min}$	0.735	0.225	-28.04	0.14

4-2 Horizontal Crack and Varying Properties Perpendicular to Crack

Specification of model is illustrated in Figure 2. Achieved SIFs and crack angle is compared with the experimental results. Table 2 shows the comparison.

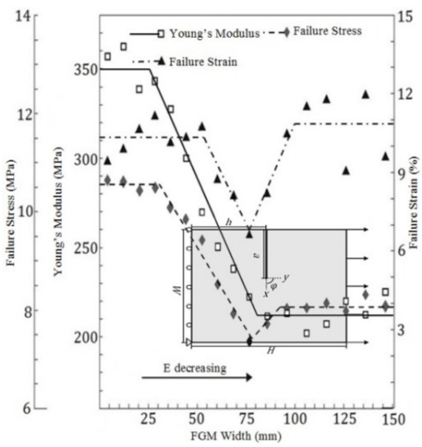

Fig. 2 Geometric dimensions of FGM plate with horizontal crack and variation of properties perpendicular to the crack

Table 2 Comparison of SIFs and crack initiation with the experimental results for FGM plate with horizontal crack and variation of properties perpendicular to the crack

Result	$K_I(MPa\sqrt{m})$	$K_{II}(MPa\sqrt{m})$	Initiation angle	Variance (°)
Exp.	0.554	0.039	0 ± 1.5	
$\sigma_{\theta\theta_{Max}}$	0.560	-0.007	1.43	1.43
$\mathcal{G}(\theta)_{Max}$	0.560	-0.007	-0.75	-0.75
$S(\theta)_{Min}$	0.560	-0.007	1.43	1.43

4-3 Oblique Crack and Varying Properties in the Crack Direction

Variation of the Young's modulus, failure stress and strain, geometric dimensions and crack orientation are displayed in Figure 3. Also, Comparison of the results with the experiments is shown in Table 3:

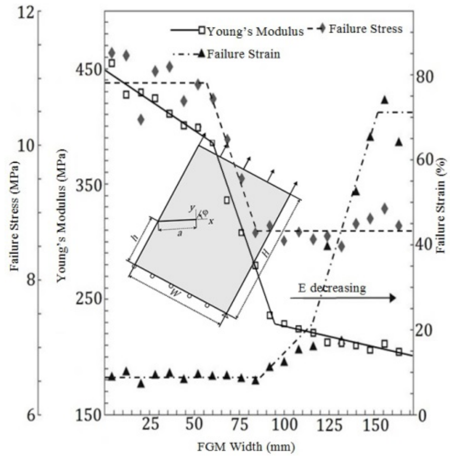

Fig. 3 Geometric dimensions of FGM plate with oblique crack and variation of properties in the crack direction

Table 3 Comparison of SIFs and crack initiation with the experimental results for FGM plate with oblique crack and variation of properties in the crack direction

Result	$K_I(MPa\sqrt{m})$	$K_{II}(MPa\sqrt{m})$	Initiation angle	Error (%)
Exp.	0.969	0.224	-19 ± 1.5	
$\sigma_{\theta\theta_{Max}}$	0.915	0.210	-23.65	24.47
$\mathcal{G}(\theta)_{Max}$	0.915	0.210	12.99	168.36
$S(\theta)_{Min}$	0.915	0.210	-22.74	19.68

5- Concluding Remarks

In this paper, FGM behavior was studied and SIFs were calculated for different crack orientation and mechanical properties, using interaction integration. Crack initiation angle was obtained from different criteria and evaluated by achieved numerical results. Results revealed that predictions of maximum tangential stress and minimum strain energy density are close, but minimum strain energy density has better accuracy. The less accuracy of the maximum strain energy release rate criterion is due to the influence of non-singular term such as T-stress has been ignored. In fact, whatever ψ is closer to zero and conditions are closer to first mode of failure, this criterion shows more precise results.