Direct Numerical Simulation of Supersonic and Transonic Compressible Viscous Flow by Kinetic Energy Preserving Scheme

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1- Introduction

Viscous and inviscid fluxes must be calculated across a cell face in the finite volume method. In calculating these fluxes, we can consider some features of the N-S equation like kinetic energy preservation (KEP). Recently, Jameson showed that the interface fluxes of a semi-discrete conservative scheme can be constructed in an alternative way which assures that the global discrete kinetic energy evolves in a manner that exactly corresponds to the true equation for kinetic energy. There is some latitude in the definition of the fluxes for such scheme (KEP), provided that the fluxes for the continuity and momentum equations satisfy a compatibility condition. The main feature of these fluxes in the KEP scheme for capturing of kinetic energy preservation lies in momentum flux estimation. This calculation method causes the deletion of convective flux terms of the kinetic energy equation. In fact, the KEP scheme uses arithmetic mean value of each pair of adjacent cells for estimation of viscous and inviscid fluxes. On the other hand, the KEP scheme conserves the total amount of kinetic energy throughout the computational domain, in addition to mass, momentum and total energy. Implementation of KEP with a fine mesh in smooth flows and flows with discontinuity (shock waves) provides a sufficiently stable condition for the calculation.

The present paper investigates the numerical solution of two-dimensional unsteady compressible Navier-Stokes equations by the kinetic energy preserving (KEP) scheme. It is introduced for solving the supersonic flow over a flat plate and transonic flow over an airfoil at low Reynolds numbers on very fine grids (with a number of cells of the order of the Reynolds number) without any artificial dissipation terms even in place of shock waves. It should be noted that the solution of the flow field with the desired scheme in this range of speed, is presented for the first time. By discretization of the governing equations based on the KEP scheme and elimination of dissipative effects, the Direct Numerical Simulation (DNS) of the flow is possible.

2- Results

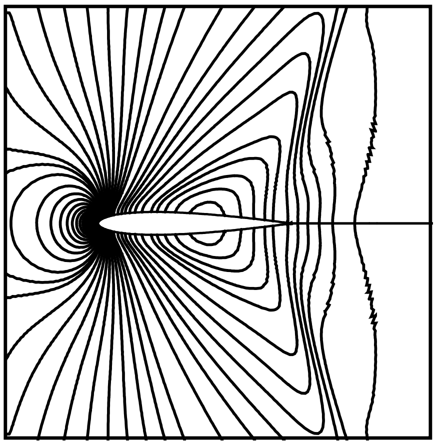
In order to evaluate the ability of KEP in the DNS simulation of compressible flow, we used this scheme for flow around a NACA 0012 airfoil. The flow conditions are shown in Table (1). In order to reduce the numerical calculations, the low Reynolds number assumption is used. The KEP results are compared with the Mittal results (Finite element computation of unsteady viscous compressible flows).

Table 1. Free stream information around the airfoil

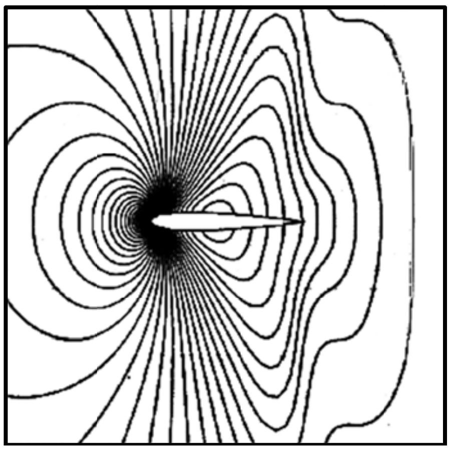
T_∞ (K)	a (deg)	$Ma_\infty = \frac{U_\infty}{c_\infty}$	$Re_\infty = \frac{\rho_\infty U_\infty c}{\mu_\infty}$
288.16	0.0	0.85	500

Pressure contours distribution

The pressure contours of KEP are compared with the Mittal results in Fig.1. The pressure contours are perfectly symmetrical above and below the airfoil because the angle of attack is zero. As can be seen, the pressure distribution of KEP (Fig.1.a) is absolutely non-oscillating in the absence of dissipative terms and generally is in good agreement with the Mittal solution (Fig.1.b).



(a)



(b)

Fig. 1. Pressure contours around NACA 0012
($Re_\infty = 500$ ، $Ma_\infty = 0.85$)
a) Present work (500×300) Grid b) Mittal results

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Pressure coefficient and skin friction variations

Fig.2 shows surface pressure coefficient distribution around the airfoil. In numerical simulation of compressible flows, the existence of artificial viscosity is essential even in smooth area (without shock waves). According to Fig.2, the KEP has been able to show the non-oscillating solution (without artificial viscosity). The pressure is in good agreement compared with the Mittal results ("numerical [8]" on Fig.2).

Fig. 3 shows the variation of skin friction on the above and bottom of the airfoil. Except for a slight increase (due to the low thickness of the boundary layer at the leading edge), skin friction decreases to the training edge of the airfoil. Generally, the results of the present numerical simulation are also in good agreement with the Mittal results.

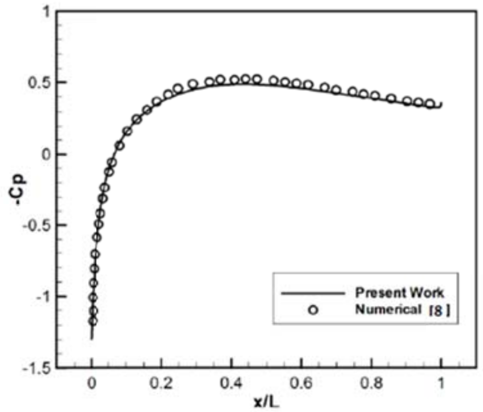

Fig. 2. Pressure coefficient distribution over NACA 0012 ($Re_\infty = 500$ ، $Ma_\infty = 0.85$)

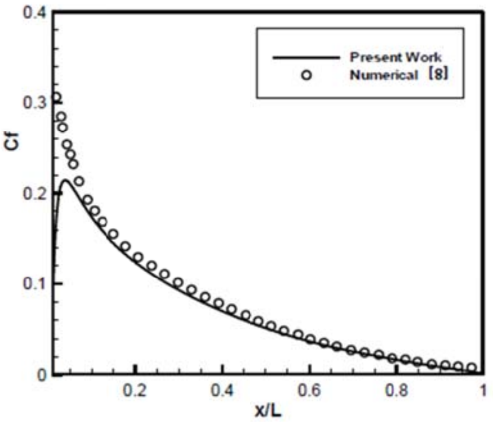

Fig. 3. Skin friction coefficient distribution over NACA 0012 ($Re_\infty = 500$ ، $Ma_\infty = 0.85$)

3- Conclusion

In the present work, viscous compressible flow equations based on the kinetic energy preserving scheme (KEP) were discretized. Then, the scheme was used for simulation of supersonic and transonic flows. The results of this study are as follows:

- The first test for supersonic laminar flow over a flat plate at Reynolds number of 1000 and Mach number 4 showed that the KEP can capture shock waves without adding artificial dissipation even at the place of shock waves.
- The second case (transonic flow over a NACA 0012 airfoil at Reynolds number 500 and Mach number 0.85) showed non-oscillatory pressure contours and skin friction coefficient distribution, even in the absence of dissipation terms.
- According to the results we can say that the KEP method is capable of presenting non-oscillating and stable solution without any artificial dissipation terms, even in the vicinity of shock waves. So, KEP can be used for direct numerical simulation of compressible flows without any worry about deletion of some turbulent flow characteristics (due to the presence of artificial dissipation terms).