The effect of Piezoelectric Patches on the Vibration and the Buckling of Euler-Bernoulli Beam under Axial Load

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1- Introduction

Recently, by improvement of technology using piezoelectric materials to increase load capacity, the enhancement of beam characterization has gained great attention.

In this paper two piezoelectric patches have been attached to an Euler-Bernoulli beam in order to investigate the effects of the position and the length of piezoelectric actuator and also the effects of boundary conditions on natural frequency, vibration behavior and mechanical stability of the beam under axial force. The patches have been attached symmetrically to upper and lower surfaces of the beam. The derived equation of motion is solved and the vibration behavior and the effects of different parameters like axial load, applied voltage, position and length of piezoelectric patches on natural frequencies and the critical load of the beam are investigated.

2- The Mathematical model of Euler-Bernoulli beam with piezoelectric actuator

An Euler-Bernoulli beam under axial load with two piezoelectric patches attached symmetrically to it is shown in Fig.1.

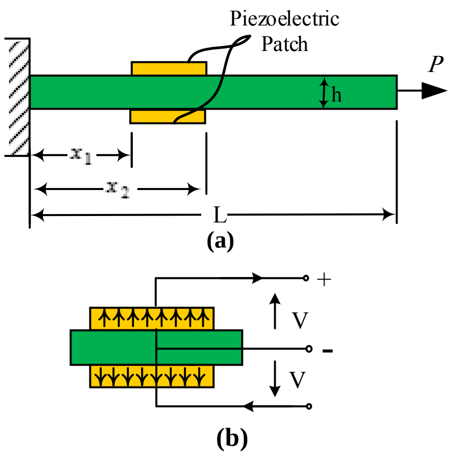

Fig. 1. Euler-Bernoulli beam (a) Clamped beam with piezoelectric actuator (b) Electric circuit of piezoelectric layer

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By applying electric voltage to the piezoelectric patches the resultant applied axial force can be calculated by the following formula:

$$F = P + F_p\left\{H(x - x_1) - H(x - x_2)\right\} \tag{1}$$

By placing Eq.4 in Eq.3 and the proposed the answer as $w(x,t) = W(x)e^{i\omega t}$, the equation of motion would be as follows:

$$EI\frac{d^4W(x)}{dx^4} - \rho A\omega^2 W(x) - \left(P + F_p\left\{H(x - x_1) - H(x - x_2)\right\}\right)\frac{d^2W(x)}{dx^2} = 0 \tag{2}$$

Since the above equation could not be solved analytically, the Galerkin method is used. The answer of the equation is supposed to be as follows:

$$W(x) = \sum_{i=1}^{n} c_i\phi_i(x) \tag{3}$$

Where, $\phi_i(x)$ are comparative functions and satisfy the boundary conditions of the equation. c_i $(i = 1 \rightarrow n)$ are unknown constants obtained by solving the characteristic equation. Placing the supposed answer (3) into the equation of motion and using the Galerkin method leads to:

$$EI\sum_{i=1}^{n}\int_0^L \phi_j(x)\frac{d^4\phi_i(x)}{dx^4}dx - P\sum_{i=1}^{n}\int_0^L \phi_j(x)\frac{d^2\phi_i(x)}{dx^2}dx - F_p\sum_{i=1}^{n}\int_{x_1}^{x_2} \phi_j(x)\frac{d^2\phi_i(x)}{dx^2}dx - \rho A\omega^2\sum_{i=1}^{n}\int_0^L \phi_j(x)\phi_i(x)dx = 0 \tag{4}$$

This equation can be written in matrix form as follows:

$$\left([K] - \omega^2[M]\right)\{C\} = 0 \tag{5}$$

The above equation is the characteristic equation of the system. ω is the natural frequency of the system and $[K]$ and $[M]$ are stiffness and mass matrices. In order for the homogeneous algebraic equation (5) to have nontrivial and nonzero answers, the determinant of the coefficient matrix should be zero. So, the characteristic equation of the system is as follows:

$$\det([K] - \Omega_n^2[M]) = 0 \tag{6}$$

Solving the above equation results in the obtained natural frequencies of the system. The natural frequencies are functions of not only the dimensions and the mechanical properties of the beam but also the magnitude of the axial force and the applied voltage of the piezoelectric actuators.

3- Numerical Results

In this section, the numerical analysis of vibration behavior and mechanical stability of the Euler-Bernoulli beam with two piezoelectric patches attached symmetrically to its upper and the lower surfaces is carried out.

Fig.2 shows changes of the frequency ratio against piezoelectric patches length. As it is seen, by increasing piezoelectric patches length, the effect of the patches on natural frequencies is increased.

Piezoelectric patches which are shorter than 20% of the beam's length have an ignorable effect on the natural frequencies. By increasing the length of the patches, their effect on natural frequencies increases and finally after a certain length ratio (80%) the effect tends to reach a constant value.

Fig.3 shows the effect of the position of the piezoelectric patches on the natural frequency ratio of the beam. As the patches approach the clamped end of the beam, the patches effect on the resultant stiffness of the structure decreases and it has less effect on the natural frequencies of the structure. In addition, piezoelectric patches have more effect on the first natural frequency of the beam than other natural frequencies.

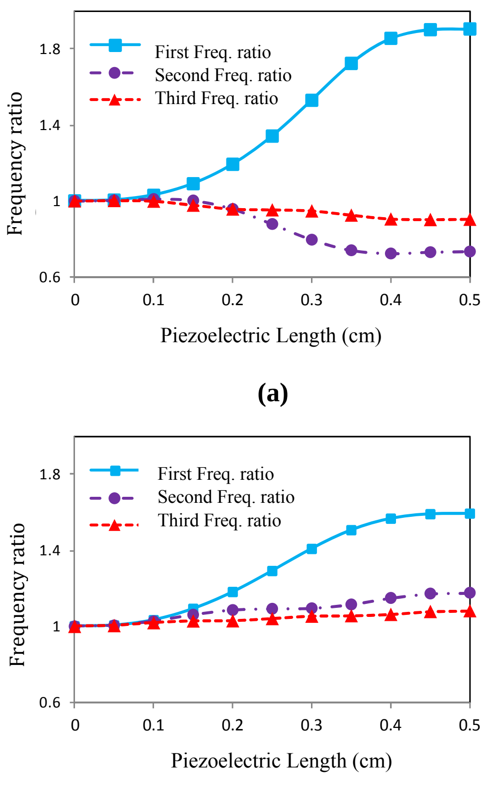

Fig.2 The effect of piezoelectric actuator length on frequency ratio of the (a) Cantilever beam (b) Simply supported beam

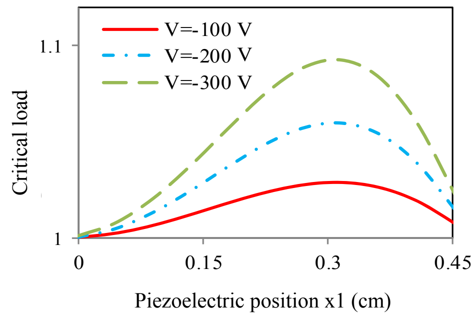

Fig.4 The Effect of piezoelectric actuator position on critical load ratio of the clamped beam

Fig.4 shows the effect of the position of the 4.5 mm long piezoelectric patches on the critical load ratio of the beam. The results show that with fixed actuator length and applied voltage, as the position of the patches approaches the free end of the clamped beam, the effect of the piezoelectric patches on the beam critical load increases.

According to the results, piezoelectric patches at 0.4 cm from the clamped end of the beam have the greatest effect on the natural frequency of the beam.

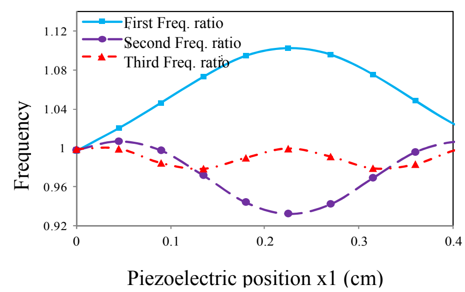

Fig.3 The effect of piezoelectric actuator position on the frequency ratio of the clamped beam

4- Conclusion

In this paper, using analytical model, the effects of two symmetrically attached piezoelectric actuators on the vibration behavior of an Euler-Bernoulli beam is investigated. The results shows that the applied voltage, the position and the length of the actuator as well as the piezoelectric actuator material have effects on natural frequencies of the beam. Setting optimizes the position and the length of piezoelectric actuators. By applying minimum electric voltage, changes in the natural frequencies and the critical load are maximized. This method is one of the applied methods for increasing load capacity and making changes in the natural frequencies in order to avoid resonance areas in structures under forced vibrations.