# The Electric Response of Piezoelectric Beam Using Dynamic Stiffness Method

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## 1. Introduction

The process of extracting energy from the environment and converting it into usable electrical energy is called energy harvesting. With the recent advances in technology in everyday life, the demand for portable and low-consumption electronic devices is increasing. Among the various mechanisms for converting mechanical energy into electricity (such as electromagnetic, piezoelectric, and electrostatic), the piezoelectric mechanism due to its ease of use, having the inherent property of electromechanical connection and high power density, received the most attention. There are a lot of data and review articles on this subject. Energy harvesting sources can be categorized from two perspectives. The first view is to consider the source of this energy, which can be human and environmental, and the second view is the type of energy such as kinetic and thermal energy that is converted into electricity.

The sources can be classified into three types: radiant, thermal, and mechanical. Recently, however, biochemical sources including glucose and fueling reactions have been added to the list. The electric field includes centralized parameters such as resistance and capacitor. So the only requirement is to obtain centralized parameters expressing the mechanical field. For this reason, modeling a degree of freedom is a convenient approach to modeling in which mechanical equilibrium and electrical ring equations are interconnected through structural equations. The one-degree-of-freedom model can be based on centralized parameters used in the study of energy conversion mechanisms for micro electromechanical systems. An extensive review of common energy harvesting technologies and energy sources reveals that higher potential power densities are obtained from piezoelectric. In this study, the focus is on modeling and analysis of one of the most common means of mechanical energy harvesting, namely a piezoelectric beam. The mentioned beam is in two shapes and has a uniform cross section and its analysis is done by dynamic hardness method. Also, with the help of this method, the maximum output voltage is obtained by finding the best place for the concentrated mass.

## 2. Dynamic Stiffness Method

The dynamic stiffness method is a powerful method for deriving the frequency response function of a structural element with a uniform cross section and combining such elements. The dynamic stiffness matrix of an element is based on the exact solution of the wave equation, so it requires fewer elements than the finite element method to analyze uniform beams, which results in a more accurate solution for high excitation frequencies. Also, the dynamic stiffness matrix of a beam can be used to model beams with different boundary conditions and to combine beams with uniform cross-section, which is a great advantage of this method over the modal analysis method.

Dynamic hardness and modal analysis methods are shown for Figure 1. The uniform configuration of the bimorph is modeled as a uniform beam. Since commonly involved piezoelectric energy pickers are often designed and manufactured as thin beams, and most bimorph beams are thin structures, it seems reasonable to assume that the beam is the Bernoulli Euler type. The piezoelectric layers and the ground are well bonded, and the electrodes that extend along the entire length of the beam are flexible and have a negligible thickness compared to the thickness of the entire beam.

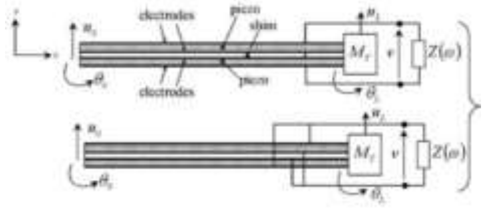


**Figure 1. Bimorph beam of energy harvesting in series and parallel connection**

## 3. Results

The damping ratios for the first two modes are $\zeta_1 = 0.0166$ and $\zeta_2 = 0.0107$ concentrated mass values $M_T = 0.5m_b$, respectively.

Using MATLAB software, programming to obtain electrical and mechanical answers for dynamic stiffness and modal analysis has been achieved. In the modal analysis method, one to five modes are used for solving. According to results, modal analysis with a solved mode, a good approximation of the frequency response function (FRF) obtains the maximum output voltage in terms of base acceleration in the first frequency range, which differs by 1.4% from the dynamic stiffness method. The resonant frequency in the dynamic hardness method is 73.77Hz, but in the modal analysis method it is 73.98Hz. Also, the diagram of the modal analysis method after the excitation frequency of 150Hz is different from the diagram of the dynamic hardness method. In the case of modal analysis with two modes, according to Figure 2, around the second peak, a good approximation of the output voltage in terms of base acceleration is obtained, which is 2.26% different from the dynamic stiffness method. Similarly, by solving the modal analysis method with more modes, the difference between the diagrams obtained from the dynamic stiffness method and the modal analysis method is reduced.

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To find the effect of centralized mass displacement, the codes written in MATLAB software are executed for the distance from the center of mass to the base of the arrow. This is because the mass transfer focuses on natural frequencies and this affects the values of damping coefficients. In this case, the analysis is examined in the immortal state.

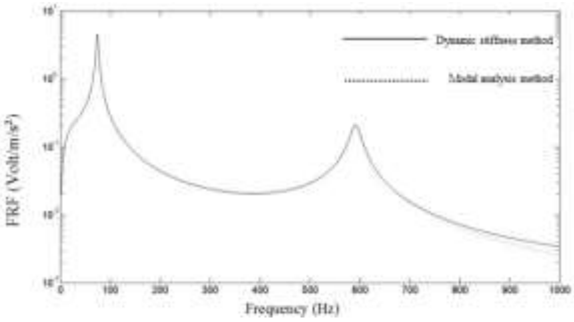


**Figure 2. FRF Voltage to acceleration ratio by two methods of dynamic stiffness and modal analysis method with the effect of two frequency modes**

The results show changes in the first natural frequency in terms of the distance of the concentrated mass from the base. Accordingly, the farther the concentrated mass is from the base, the lower the natural frequency of the first. The first natural frequency for concentrated mass at a distance of 0.5 mm from the base of the beam is 127.8Hz, which gradually decreases to a distance of 15 mm from the base, decreasing the first natural frequency with a very slow slope.

In connection with changes in the natural frequency of the second beam relative to the distance of the concentrated mass from the base, as in the first natural frequency, the natural frequency decreases at a very slow slope at first, but with increasing mass distance from the base, this process occurs rapidly. According to results, the second lowest natural frequency at a distance of 26 mm is the concentrated mass from the moving base and the value is 560.1Hz. According to results, it is shown that the second natural frequency has an intermittent relationship with the concentrated mass distance, during which the second natural frequency value is 771.6Hz at 4mm and 46mm mass distances from the beam.

Because most of the vibrations that can be picked up from the environment occur in the low frequency domain, the low frequency of the beam intensification can be a great help to the amount of energy harvested. Therefore, regardless of the amount of energy harvested, the installation of a concentrated mass at the end of the beam can be useful for such consumption.

The maximum power output is a determining parameter in concentrated mass displacement. Therefore, for the first two modes, the maximum ratio of the output voltage to the ground acceleration in terms of the distance from the concentrated mass to the ground is shown in Figure 3. According to the figure, it is clear that the maximum voltage-to-acceleration ratio of the base occurs around the first resonant frequency for a distance of 38 mm from the concentrated mass to the base, which is $8.622\frac{\mathrm{volt}}{\mathrm{m/s^2}}$. Moreover, there is the lowest ratio of output voltage to base acceleration at the closest point of concentrated mass to base and its value is $6.125\,\frac{\mathrm{volt}}{\mathrm{m/s^2}}$.

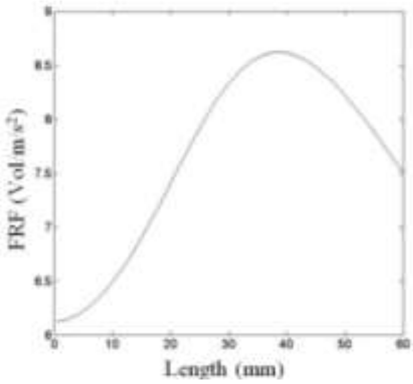


**Figure 3. Output voltage ratio on base acceleration at first resonant frequency in terms of distance from concentrated mass to beam base**

## 4. Conclusion

In this study, a dynamic stiffness method was developed for a single beam engaged with a uniform cross section with a concentrated mass. This method is based on accurate solution. Newton's approach was used to obtain the relationships in which the output voltage amplitude was a function of the slope at the beginning and end of the beam and these two parameters were obtained with the help of the dynamic stiffness matrix of the beam. Moreover, the electrical output was obtained by using the modal analysis method for a mode up to five modes. They kept their distance. One of the main aims of this research was to investigate the effects of focused mass displacement on electrical output and natural frequencies of the system in the immortal state by dynamic stiffness method. Results showed that systems whose concentrated mass was closer to the base had a larger natural first frequency and the lowest natural first frequency was obtained for the concentrated mass at the tip of the beam. But the changes of the second natural frequency were periodically related to the distance of the concentrated mass from the base, so that the highest value of the second natural frequency for the mass position at 46 mm was obtained from the base, and the lowest for the concentrated mass at 26 mm from the base. Then the ratio of output voltage to base acceleration for concentrated mass at different distances was investigated. The highest output at the first resonant frequency was obtained for the concentrated mass at a distance of 38 mm from the base, but the highest output at the second resonant frequency was obtained for a system with a concentrated mass at a distance of 16 mm from the base. Therefore, if the maximum output voltage needs to be removed, 16mm or 38mm concentrated mass distances from the base were suggested.