# Nonlinear Control of Motion of a Spherical Robot on Inclined Surfaces Based on Feedback Linearization Method 

Mohamad Aalipour ${ }^{1}$ Ali Mokhtarian ${ }^{2}$

Hasan Karimpour ${ }^{3}$

## 1. Introduction

The spherical robot is referred to as spherical mobile robots equipped with an internal actuating mechanism that moves on the ground due to the rolling of their external shell. These robots have special benefits compared to other mobile robots. They have only one contact point with the surface. This point allows robot to move with minimum friction and the least energy. Their spherical shell provides a proper space to embed the internal components and protects them from damage and dust. Hence, this robot is often used in space research and exploration of military and unknown areas. To date, extensive researches have been carried out on the design, simulation and control of spherical robots.
In the present study, the two-dimensional motion of a spherical pendulum robot on an inclined surface is controlled. In this regard, first, dynamic modeling and extraction of two-dimensional motion equations of pendulum-driven spherical robot on an inclined surface, then, designing a nonlinear controller and simulation of robot motion in MATLAB software are performed. Furthermore, considering the parametric and structural uncertainties for the system, the performance of controller is evaluated. It should be noted that due to the lack of number of actuators in this robot, a nonlinear controller based on a special feedback linearization method is designed.

## 2. Structural Model of Spherical Robot

Spherical robot studied in this paper is pendulum type. The main components of this spherical robot are the motor, the external shell, the internal components of the robot, and the pendulum (Figure 1). The motor is located at the joint between the pendulum and the internal components of the robot, and by creating the rotations for the pendulum in two directions and changing the position of the center of mass of the spherical robot, the robot moves in forward and lateral directions.


Figure 1. Structure of a spherical pendulum robot
However, in this study, only the robot's forward motion without considering the possibility of lateral motion, using the pendulum rotation in one direction during a two-dimensional motion, has been analyzed and controlled.

## 3. Modeling and Designing of Controller

To dynamic analysis, the two-dimensional motion of a spherical robot on an inclined surface is considered (as shown in Fig. 2). According to Figure 2, $\alpha$ is the shell rotation angle, $\varphi$ is the pendulum rotation angle, and $\gamma$ is the slope angle of inclined surface.


Figure 2. Moving the spherical robot on inclined surface
We consider $\alpha$ and $\varphi$ as the system generalized coordinates and the equations of motion of the robot are derived in terms of these two coordinates and their derivatives. The matrix form of the motion equations can be expressed in the form of Equation (1).
$\left[\begin{array}{ll}\mathrm{M}_{11} & \mathrm{M}_{12} \\ \mathrm{M}_{21} & \mathrm{M}_{22}\end{array}\right]\left[\begin{array}{l}\ddot{\alpha} \\ \ddot{\varphi}\end{array}\right]+\left[\begin{array}{l}\mathrm{N}_{1} \\ \mathrm{~N}_{2}\end{array}\right]=\left[\begin{array}{l}\tau \\ \tau\end{array}\right]$
where, $\tau$ is torque of motor. Given the main control variable of the robot, i.e., $\alpha, \tau$ can be expressed as Equation (2).

$$
\begin{equation*}
\tau=\left(\frac{M_{11} M_{22}-M_{12} M_{21}}{M_{22}-M_{12}}\right) \ddot{\alpha}+\frac{M_{22} N_{1}-M_{12} N_{2}}{M_{22}-M_{12}} \tag{2}
\end{equation*}
$$

[^0]The control torque should be applied to the robot is considered as Equasion (3).

$$
\begin{align*}
& \tau=\left(\frac{M_{11} M_{22}-M_{12} M_{21}}{M_{22}-M_{12}}\right)\left(\ddot{\alpha}_{d}-k_{v}(\dot{\alpha}-\right. \\
& \left.\left.\dot{\alpha}_{d}\right)-k_{p}\left(\alpha-\alpha_{d}\right)\right)+\frac{M_{22} N_{1}-M_{12} N_{2}}{M_{22}-M_{12}} \tag{3}
\end{align*}
$$

where, $\alpha_{d}$ represents the desired time function of rotational motion of the sphere shell, $k_{v}$ and $k_{p}$ are respectively, the positive derivative and proportional control coefficients of the system. By applying this control torque to the robot, control equation of system is determined as Equation (4).

$$
\begin{equation*}
\ddot{\mathrm{e}}+\mathrm{k}_{\mathrm{v}} \dot{\mathrm{e}}+\mathrm{k}_{\mathrm{p}} \mathrm{e}=0 \tag{4}
\end{equation*}
$$

where, $\mathrm{e}=\alpha-\alpha_{\mathrm{d}}$ is the time function of the shell rotation error angle relative to the desired condition. By selecting positive values for the control coefficients, the response of Equation (4) exponentially converges to zero.

## 4. Simulation Results

In this section, the numerical simulation of the controlled motion of a spherical robot on an inclined surface is conducted by considering a desired time function for the rotational motion of the robot shell as Equation (5).
$\alpha_{d}=\sin (0.2 t)$
Assuming two percent error for the values of all the physical parameters of the robot, applying the disturbance torque 1.5 N.M to the spherical shell, and selecting the undesirable initial conditions corresponding to the rotation angle 0.5 rad , we examined the performance of the controller in creating a resistant response that follows the predetermined desirable trajectory. Based on the type of controller function and the characteristics of the transient error response and considering the saturation limit of the actuator, for $\mathrm{k}_{\mathrm{v}}$ and $\mathrm{k}_{\mathrm{p}}$, the value of 20 is selected using the trial and error method. The simulation results in this case, including the shell rotation angle diagram and error angle diagram, are presented in Figures 3 and 4.


Figure 3. Rotation angle of the spherical robot in comparison with the ideal angle


Figure 4. Diagram of spherical shell rotation error relative to the desired condition

As shown in Figures 3 and 4, in spite of presence of parametric and structural uncertainties for system, the rotation angle of the spherical robot has been controlled as it follows its ideal time function with a small permanent error. It is necessary to mention the required torque of actuator during movement of robot has not exceeded the assumed saturation limit.

## 5. Conclusion

In this study, dynamic modeling and planar motion control of a spherical pendulum robot on an inclined surface were performed. For this purpose, first, the planar motion of robot was modeled and by selecting two generalized coordinates such as shell rotation angel ( $\alpha$ ) and pendulum rotation angle $(\varphi)$, the dynamic equations of the two-dimensional motion of robot on inclined surface were derived using Lagrange method. Due to the lack of number of actuators compared to the number of degrees of freedom of the spherical robot, controlling the robot was challenging in establishing stability. Therefore, a nonlinear controller was designed using a feedback linearization method. Then, considering the initial conditions that did not conform the trajectory, the parametric uncertainty as well as the disturbance torque on the system, the controller resistance was evaluated in presence of these uncertainties. The results were obtained in such unstable conditions indicated the proper performance of the controller in following the desired time trajectories determined for the rotation angle of the spherical shell while moving on the inclined surface. Furthermore, the design of this nonlinear controller makes it possible to use a motor with the torque under saturation limit for the robot. This issue (torque saturation limit of actuator) is one of the most important limitations in the design of spherical robots.


[^0]:    1. Mohammad Aalipour, MSc graduate, Department of Mechanical Engineering, Khomeinishahr Branch, Islamic Azad University, Khomeinishahr/Isfahan, Iran.
    2. Corresponding Author: Ali Mokhtarian, Assistant professor, Department of Mechanical Engineering, Khomeinishahr Branch, Islamic Azad University, Khomeinishahr/Isfahan, Iran.
    Email: mokhtarian@iaukhsh.ac.ir
    3. Hossein Karimpour, Assistant professor, Department of Mechanical Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran.
